



MACHAKOS UNIVERSITY

University Examinations for 2019/2020 Academic Year

SCHOOL OF BUSINESS AND ECONOMICS

DEPARTMENT OF ECONOMICS

FIRST YEAR FIRST SEMESTER EXAMINATION FOR

BACHELOR OF ECONOMICS AND FINANCE

BACHELOR OF ECONOMICS

EES 100: MATHEMATICS FOR ECONOMISTS 1

DATE: 11/12/2019

TIME: 2.00-4.00 PM

INSTRUCTIONS:

- (i) Answer question one (Compulsory) and any other two questions
- (ii) Show your working clearly

QUESTION ONE (COMPULSORY) (30 MARKS)

The economy of a given country depends on three main sectors namely; Agriculture (A), Tourism (T) and Industry (I). Agriculture consists of two main elements namely; Livestock production (L) and Crop production (C). Tourism consists of three main elements namely; Wildlife (W), Scenery (S) and Recreation (R). Industry consists of two main elements namely; Motors (M) and Physical environment (P) whose contribution to the economy is the same as that of Wildlife (W), implying that $W = P$.

a) Given that the distributive law is given as

$$A \cup (T \cap I) = (A \cup T) \cap (A \cup I) \text{ and}$$

$$A \cap (T \cup I) = (A \cap T) \cup (A \cap I),$$

represent the above information in form of three main sets, that is, A, T and I. Also verify the distributive law (7 marks)

b) An economy is defined by:

$$Y = C + I + G + X - M$$

$$C = c_0 + c_1Y$$

$$I = i_0 + i_1Y$$

$$M = m_0 + m_1Y$$

$$G = G_0$$

$$X = X_0$$

- i. What is the difference between c_0 and m_0 on the one hand, and c_1 and m_1 on the other hand? (2 marks)
- ii. Find the value of Y in terms of c_0 , c_1 , m_0 , m_1 , i_0 , i_1 , X_0 , and G_0 , and denote the value by \bar{Y} (4 marks)
- iii. Find the value of C in terms of c_0 , c_1 , m_0 , m_1 , i_0 , i_1 , X_0 , and G_0 , and denote the value by \bar{C} . (4 marks)

c) The commodity and money markets for an economy are defined by the following equations:

Commodity Market

$$Y = C + I$$

$$C = 200 + \frac{2}{5}Y$$

$$I = 1900 - 12r$$

Money Market

$$M_{DT} = \frac{1}{2}Y$$

$$M_{DS} = 100 - 10r$$

$$M_S = 1500$$

- (i) Derive the IS and the LM functions for the economy (5 marks)
- (ii) Determine the equilibrium income and the rate of interest for the economy (4 marks)

(d) You are given the following supply and demand functions for a market:

Demand function

$$0.04P^2 + 0.4P + 0.2Q = 20$$

Supply function

$$0.15P^2 + 2P + 0.5Q = -50$$

- i. Determine the price elasticity of demand at $P = 10$ and comment on your results (2 marks)
- ii. Determine the price elasticity of supply at $P = 20$ and comment on your results

(2 marks)

QUESTION TWO (20 MARKS)

- a) An economy is described by the following production function:

$$Q = \frac{K^2 - KL + L^2}{L^3}$$

Demonstrate Eulers theorem

(3 marks)

- b) You are given the following information about a firm

$$P = 4 - \frac{1}{4}Q$$

$$ATC = 0.05Q^2 - 0.3Q + 2 + \frac{4}{Q}$$

- i. Find the output level which will maximize the profits of the firm (4 marks)
- ii. Find the output level that will maximize the profits of the firm if a lump tax of K is imposed, K being a constant (4 marks)
- iii. Explain the outcome in (ii) above (1 mark)
- c) The demand and average total costs facing a firm are given below:

$$P = 6 - \frac{3}{5}Q$$

$$ATC = Q - \frac{2}{5} + \frac{3}{Q}$$

- i. Find Q when profits equal to zero (4 marks)
- ii. Determine the level of Q that corresponds to a profit level of -3 (4 marks)

QUESTION THREE (20 MARKS)

- a) The total cost function facing a firm is:

$$TC = 50 + 20Q - Q^2 + 0.5Q^3$$

- i. Determine the AFC, ATC, AVC and MC (4 marks)
- ii. Find the level of Q where the slopes of the MC function and AVC function will be equal to zero (4 marks)
- b) The demand function for a commodity is

$$P = 50 - 0.5q$$

The cost of producing the commodity is made up of fixed cost of 200 shillings and variable cost of 0.2 shillings per unit.

- (i) Find the profit function for the commodity (4 marks)
- (ii) Determine the output level at which $\frac{d\pi}{dQ} = 0$ (3 marks)
- c) The demand functions for two products are:

$$Q_{d1} + P_1 = \frac{1}{4}P_2 + 8$$

$$Q_{d2} + \frac{1}{2}P_2 = \frac{3}{4}P_1 + 8$$

Where Q_{d1} and Q_{d2} are quantities of goods 1 and 2 demanded respectively, and P_1 and P_2 are the corresponding prices of the goods. If the quantity demanded of each good is 10, find the price at which each good will be sold. (5 marks)

QUESTION FOUR (20 MARKS)

a) The utility of a consumer is defined by

$$U = Q_1^2 Q_2^2$$

Where U is the level of utility and Q_1 and Q_2 are the quantities of commodity 1 and commodity 2 consumed.

- i. The consumer initially purchases 5 units of Q_1 and 2 units of Q_2 . If he purchases 4 units of Q_2 , how many units of Q_1 will he purchase to maintain the same level of utility? (3 marks)
- ii. If he should decide to purchase 3 units of Q_1 , how many units of Q_2 will he purchase to maintain the same original level of utility? (3 marks)

b) The demand for commodity “a” is expressed as a function of some related commodity “b” as:

$$Q_a = 7 + (P_b)^{-\frac{1}{4}}$$

- i. Find the level of demand for “a” when the price of “b” is 81. (2 marks)
- ii. Find the cross-elasticity of demand for “a” with respect to price of “b” when $P_b = 16$. Interpret your results (4 marks)

c) A Cobb-Douglas production function is defined by

$$Q = AK^\alpha L^{1-\alpha}$$

- i. Find marginal product of capital and marginal product of labor (2 marks)
- ii. Determine the corresponding APL and APK (4 marks)
- iii. Express the APL and the APK as a function of capital to labor ratio (2 marks)

QUESTION FIVE (20 MARKS)

a) An open economy is defined by the following model

$$Y = C + I + G + X - M$$

$$C = c_0 + c_1 Y^d$$

$$T = t_0 + t_1 Y$$

$$M = m_0 + m_1 Y$$

$$I = I_0$$

$$G = G_0$$

$$X = X_0$$

- i. Find the equilibrium income (3 marks)
 - ii. Find the equilibrium imports (2 marks)
 - iii. What is the effect of a change in t_0 on \bar{Y} and a change in m_0 on \bar{Y} ? (2 marks)
 - iv. Determine the export multiplier for the system (1 mark)
- b) The supply and demand functions for a firm are given by
- $$Q_d = 60 - \frac{1}{3}P$$
- $$Q_s = -30 + \frac{2}{3}P$$
- If the government decides to impose a per unit tax t on quantity supplied, find:
- i. The tax rate that will maximize government tax revenue (4 marks)
 - ii. The maximum tax revenue, assuming tax function to be $T = tQ$ (2 marks)
- c) Clearly explain the need of Mathematics in Economics (6 marks)