



# MACHAKOS UNIVERSITY

University Examinations for 2019/2020 Academic Year

SCHOOL OF BUSINESS AND ECONOMICS

DEPARTMENT OF ECONOMICS

THIRD YEAR FIRST SEMESTER EXAMINATION FOR

BACHELOR OF ECONOMICS AND FINANCE

BACHELOR OF ECONOMICS AND FINANCE

BACHELOR OF ECONOMICS

BACHELOR OF ARTS

EES 300: MATHEMATICS FOR ECONOMISTS III

DATE: 29/11/2019

TIME: 8.30-10.30 AM

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## INSTRUCTIONS:

- (i) Answer question one (Compulsory) and any other two questions
- (ii) Do not write on the question paper
- (iii) Show your working clearly

## QUESTION ONE (COMPULSORY) (30 MARKS)

- a) The marginal cost of a firm is given by the following function

$$C'(Q) = 50e^{0.5Q} + 10. \quad \text{The fixed cost } C_F = 300$$

- i. Find the total cost function  $C(Q)$  (3 marks)
  - ii. Find the average cost function  $AC(Q)$  (3 marks)
  - iii. What is the total cost when  $Q=20$ ? (3 marks)
- b) The growing value ( $V$ ) of wine is a function of time as shown below

$$V = K \exp\left(t^{\frac{1}{2}}\right)$$

Assume that wine is already paid for and storage costs are nonexistent.

When should the owner sell the wine to maximize profits when rate of discount,  $r$  is 0.01.

(Hint: find the present value of  $V$  by discounting  $V$  to its present value first) (5 marks)

- c) Use Jacobian determinant to test the existence of functional dependence between the pair of functions given below (4 marks)

$$y_1 = 2x_1 + 3x_2$$

$$y_2 = 4x_1^2 + 12x_1x_2 + 9x_2^2$$

- d) Verify that each of the following differential equations is exact and solve each by the four-step procedure (6 marks)
- $2yt^3 dy + 3y^2 t^2 dt = 0$
  - $t(1 + 2y)dy + y(1 + y)dt = 0$
- e) Find the following (6 marks)
- $\int \left(3e^x + \frac{3}{x^2}\right) dx$  ( $x \neq 0$ )
  - $\int x \ln x dx$  ( $x > 0$ )

### QUESTION TWO (20 MARKS)

- a) You are given the following demand and supply functions

$$Q_d = \alpha + \beta P^2 + \sigma \frac{dP}{dt} \quad \alpha, \beta, \sigma > 0$$

$$Q_s = -\gamma + \delta P \quad \gamma, \delta > 0$$

Assuming that the rate of change of price over time is directly proportional to the excess demand, find the time path  $P(t)$ , general solution (8 marks)

- b) Find the time path of capital  $K(t)$ , given the following rates of capital formation and initial capital stocks

$$\text{i. } I(t) = 8t^{\frac{2}{3}} \quad K(0) = 200 \quad (3 \text{ marks})$$

$$\text{ii. } I(t) = 5t^{\frac{1}{4}} \quad K(0) = 210 \quad (3 \text{ marks})$$

- c) Consider the following function

$$Z = -x^2 + xy - 2y^2 + 6x + 4y$$

- Find the critical values of  $x$  and  $y$  and the stationary point (3 marks)
- By applying Hessian matrix  $H$ , determine whether the stationary point presents a maximum or a minimum (3 marks)

### QUESTION THREE (20 MARKS)

- a) Solve the following difference equations using iterations

$$\text{i. } Y_{t+1} = Y_t + 3 \quad Y_0 = 9 \quad (3 \text{ marks})$$

$$\text{ii. } Y_{t+1} - 0.5Y_t = 6 \quad Y_0 = 13 \quad (4 \text{ marks})$$

$$\text{iii. } Y_{t+1} = Y_t - 4 \quad Y_0 = 9 \quad (4 \text{ marks})$$

- b) For the following demand and supply functions, find the intertemporal equilibrium price and determine whether the equilibrium is stable

- i.  $Q_{dt} = 15 - 3P_t$   $Q_{st} = 4 + 2P_{t-1}$  (3 marks)  
 ii.  $Q_{dt} = 14 - 3P_t$   $Q_{st} = -6 + P_{t-1}$  (4 marks)  
 iii.  $Q_{dt} = 15 - 3P_t$   $Q_{st} = +2P_{t-1} - 5$  (4 marks)

#### QUESTION FOUR (20 MARKS)

- a) Consider the following market model with inventory:

$$Q_{dt} = 14 - 4P_t$$

$$Q_{st} = -6 - 3P_t$$

$$P_t = P_t - 0.4(Q_{st} - Q_{dt})$$

Find the time path  $P_t$  and determine whether it is convergent (6 marks)

- b) Find the consumer surplus given the following demand function (6 marks)

$$Q_d = 25 - \frac{1}{3}P \text{ and Quantity supply given as } Q_s = -1 + \frac{1}{2}P$$

- c) Suppose  $(t) = 5t^{\frac{1}{3}} + 2$ , what will be the capital formation between year 2 and year 6? (4 marks)

- d) The growing value of GNP is given by:

$$GNP_t = GNP_0 e^{rt} \quad r = 1.5\%$$

- i. If  $GNP_0 = 500$ , find the value of GNP 10 years from now (2 marks)  
 ii. If  $GNP_0 = 1000$ , after how many years will the GNP double? (2 marks)

#### QUESTION FIVE (20 MARKS)

- a) Find Y and r by Cramer's rule and inverse matrix, given the following IS-LM models (8 marks)

Goods Market

$$Y = C + I$$

$$Y = 200 + 0.2Y$$

$$I = 8 - 0.3r$$

Money Market

$$Md = 100 + 0.5Y + 0.25r$$

$$Ms = 120$$

$$Md = Ms$$

- b) Find the following

i.  $\int_2^3 \frac{12x}{6x^2+2} dx$  (3 marks)

ii.  $\int (x+3)x + 1^{\frac{1}{3}} dx$  (3 marks)

- c) If the population grows according to the function  $H = H_0(2)^{bt}$  and consumption by the function  $C = C_0 e^{at}$ , find the rates of growth of population of consumption and of per capita consumption by using the natural log (6 marks)