Control Volume Analysis of Unsteady Mhd Mixed Convection Nanofluid Flow as A Result of Stretching Surface with Suction

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Abstract

Many heat transfer processes in engineering problems in areas such as nuclear reactors and electronics, as well as in biomedicine and food industry require the knowledge of nanofluids. Consequently, investigations leading to understanding of the role played by nanofluids in heat transfer enhancement in these processes is vital. Currently, numerous studies are being conducted on nanofluids for the benefits associated with low energy costs and less negative environmental impact in industry and society. In the studies, water is commonly used as base for nanofluids in heat transfer applications due to its ability and availability for heat transport. In most of these investigations influence of nanoparticles have been analyzed to determine enhancement of energy transfer on stretched sheets. In this research, magnetohydrodynamic mixed convection flow of a nanofluid over a stretching sheet with water equally as the base fluid and either Copper or Silver as nanoparticles is examined anddiscussed. The physical problem is modeled using systems of unsteady nonlinear differential equations (DE) subject to prescribed boundary and initial conditions, which are then studied using Finite volume approach. These DE comprise of the classical continuity, momentum, concentration and energy equations, which are subsequently non dimensionalized and discretized in rectangular domain. The effect of nanoparticle volume fraction values on velocity, temperature and concentration profiles are discussed.

Key words: Nanofluid, straight stretching surface, mixed convection, finite volume method.

INTRODUCTION

In recent years, extensive research has been conducted on manufacturing materials whose grain sizes are measured in nanometers. These materials have thermal, electrical, and chemical properties, hence can be applied this emerging nanotechnology to enhance thermal energy in engineering. Nanometer-sized particles can be suspended in industrial heat transfer fluids such as water, ethylene glycol, or oil to produce a new class of engineered fluids with high thermal conductivity.

Thermal conductivities of most solid materials are higher than those of liquids, therefore thermal conductivities of particle fluid mixtures are expected to increase. Fluids with higher thermal conductivities would have potentials for many thermal management applications. Due to the very small size of the suspended particles, nanoparticle fluid mixtures could be suitable as heat

transfer   fluids in many existing heat transfer devices, including those miniature devices in which sizes of components and flow passages are small. Nanoparticles also act as a lubricating medium when they are in contact with other solid surfaces.

Nanofluid is a very active field of study especially within the engineering community and it is experiencing rapid development in research and applications worldwide, as a result of interesting features such as increased thermal conductivity. Low thermal conductivity of convectional fluids is improved when solid particles are added [6].

In this study, an unsteady MHD mixed convection boundary layer nanofluid flow as a result of stretching surface is conducted using Finite Volume Method (FVM). FVM has emerged as a powerful alternative to other numerical methods such spectral relaxation method, Rungekutta method, and backward facing step method, finite-difference method, scaling transformations, direct numerical simulation, Finite Element method, homotopy analysis method, marker-and-cell method and perturbation method have been used to solve the linear partial differential, particularly in case where better accuracy is required due to difficulties such as stress concentration or where the domain extends to infinity.

Researchers have studied properties of nanofluid, thermal conductivity of nanofluid, flow on a stretching surface and a few on finite volume approach.[12] discussed Teerapong Borirak experimental investigation of titanium nanofluids on the heat pipe thermal efficiency. The enhancement heat transfer of the heat transfer devices was done by changing the fluid transport properties and flow features of working fluids. The heat pipe was fabricated from the straight copper tube with the outer diameter of 15mm and length of 600 mm. The heat pipe with the de-ionic water, alcohol, and nanofluids) were tested. They used titanium nanoparticles with diameter of 21 nm which the mixtures of alcohol and nanoparticles were prepared using an ultrasonic homogenizer. Results showed the nanoparticles have a significant effect on the enhancement of thermal efficiency of heat pipe. [5], illustrated the frequent and wide occurrence of non-Newtonian fluid behavior in a diverse range of applications, both in nature and in technology. He used materials as foams, suspensions, polymer solutions and melts. Each type of non-Newtonian fluid behavior were illustrated via experimental data on real materials and results showed that nanofluid are good in transferring heat. *Properties* of Gold-water using molecular dynamics was explored by [7]. His study treats the case of a gold–water nanofluid at different particle volume fractions between o.o1and 0.15 volumes. He used water confined between gold nanolayers for him comprehend physical phenomena at the interface of gold and water using different plates. He treated nanofluid as ideal mixture which was proved correct. Heat transfer enhancement by using nanofluids in forced convection flows was analyzed by [14]. Uniformly heated tube and a system of parallel, coaxial and heated disks was made use of. Numerical results, as obtained for water and Ethylene Glycol mixtures showed the inclusion of nanoparticles into the base fluids produced a considerable augmentation of the heat transfer coefficient that clearly increased with an increase of the particle concentration. Ethylene glycol nanofluid offered a better heat transfer enhancement than water. [10], examined an assessment of the effectiveness of nanofluids for single-phase and two-phase heat transfer in micro-channels. Experiments were performed to explore the micro-channel cooling benefits of water-based nanofluids containing small concentrations of Aluminium oxide. The high thermal conductivity of nanoparticles was shown to enhance the single-phase heat transfer coefficient, especially for laminar flow. Higher heat transfer coefficients were achieved mostly in the entrance region of

micro-channels. However, the enhancement was weaker in the fully developed region, proving that nanoparticles have an appreciable effect on thermal boundary layer development. A room temperature enquiry consisting of water and nanofluids consisting of Silver-Aluminium as well as Silver-Copper nanoparticles was conducted by [5]. It was found that the suspensions of Silver-Aluminium nanoparticles showed enhancement in thermal conductivity slightly more than Silver-Copper nanoparticle suspensions. Also, the suspensions of carbon nanotubes in different fluids were found to possess increasing enhancement. A study on estimation of heat transfer coefficient and friction factor in the transition flow with low volume concentration of Aluminium oxide nanofluid flowing in a circular tube and with twisted tape was examined by [13]. They evaluated heat transfer coefficient and friction factor for flow in a tube and with twisted tape inserts in the transition range of flow with Aluminium oxide nanofluid. Findings showed considerable enhancement of convective heat transfer with Aluminium oxide nanofluids compared to flow with water. [1], researched on melting effect on unsteady hydromagnetic Flow of a nanofluid past a stretching Sheet. Unsteady, laminar, boundary-layer flow with heat and mass transfer of a nanofluid along a horizontal stretching plate in the presence of a transverse magnetic field, melting and heat generation or absorption effects was explored. The model used for the nanofluid incorporates the effects of Brownian motion and thermophoresis. The governing partial differential equations were transformed into a set of non-similar equations and solved numerically by an efficient implicit, iterative, finite-difference method. Numerical results for the steady-state velocity, temperature and nanoparticles volume fraction profiles as well as the time histories of the skin-friction coefficient, Nusselt number and the Sherwood number were presented graphically and discussed. A study on effect of partial slip boundary condition on the flow and heat transfer of nanofluids past stretching sheet prescribed constant wall temperature was appraised by [2]. They analyzed the development of the slip effects on the boundary layer flow and heat transfer over a stretching surface in the presence of nanoparticle fractions. In the modeling of nanofluid the dynamic effects including the Brownian motion and thermophoresis are taken into account. In the case of constant wall temperature a similarity solution was presented. The solution depends on a Prandtl number, slip factor, Brownian motion number, Lewis number, and thermophoresis number. The dependency of the local Nusselt and local Sherwood numbers on these five parameters was numerically investigated. The results showed the flow velocity and the surface shear stress on the stretching sheet and also reduced Nusselt number and reduced Sherwood number were strongly influenced by the slip parameter. Unsteady flow of a nanofluid in the stagnation point region of a time-dependent rotating sphere was investigated by [3] .The boundary layer equations were normalized via similarity variables and solved numerically. The nanofluid was treated as a two-component mixture that incorporated the effects of Brownian diffusion and thermophoresis together as two ways of slip velocity in laminar flows. [9], analyzed MHD flow of nanofluids over an exponentially stretching sheet in a porous medium with convective boundary conditions. An incompressible fluid fills the porous space and study was made for the nanoparticles namely Copper, silver, Alumina and Titanium Oxide and water as the base fluid. The non-linear partial differential equations (PDE) governing the flow were reduced to an ordinary differential equation (ODE) by similarity transformations. The obtained equations were solved for the development of series solutions and convergence of the obtained series solutions was analyzed. An inquiry on heat transfer augmentation in a differentially heated square cavity using copper–water nanofluid was carried out by [4]. [11], researched on nanofluid flow over a stretching surface in presence of chemical reaction and thermal radiation. They focused on a steady MHD boundary layer flow of an electrically

conducting nanofluid over vertical permeable stretching surface with variable stream conditions. The group theoretic method was used to simplify the governing partial differential equations. The reduced governing equations are solved using a fourth order Runge-Kutta method and Shooting techniques to predict the heat and mass transfer characteristics of the nanofluid flow. Numerical results converged.

From the foregoing literature review, what many of the researches carried out shows that MHD nanofluid flow to a stretching surface has not been explored much and the few who have ventured into stretching surface did not pay attention to a straight surface. Also different methods have been used by researchers to solve problems but in my case I will use FVM since it is convergent and bound.

DESCRIPTION AND FORMULATION

Description of problem

In the current study, consider unsteady laminar MHD mixed convective nanofluid flow as a result of straight stretching surface situated at x axis with stretching velocity $u = bx$, where b is a constant. Let suction velocity be $v = v_w$, the temperature at the wall $T = T_w$ and nanoparticle concentration at the stretching surface are $C = C_w$. The temperature of the free stream nanofluid is $T = T_\infty$ and and the ambient concentration is C_∞. The x is along the stretching surface and y direction is orthogonal to the stretching surface. The flow experiences magnetic force.

The boundary and initial conditions are $u = 0, v = 0, T = T_w, C = C_w \ at \ t < 0$.

The properties of nanofluid are defined as follows, [8]

$$\mu_{nf} = \frac{\mu_f}{(1-\Phi)^{2.5}}, k_{nf} = k_f \left(\frac{k_s + k_f + 2\Phi(k_s - k_f)}{k_s + 2k_f + \Phi(k_s - k_f)} \right), (\rho c_p)_{nf}$$

$$= (1-\Phi)(\rho c_p)_f + \Phi(\rho c_p)_s, \rho_{nf} = (1-\Phi)\rho_f + \Phi\rho_s, \Phi_0$$

$$= (1-\Phi)^{2.5} \left(1 - \Phi + \Phi \left(\frac{\rho_s}{\rho_f} \right) \right), \Phi_a = \left(1 - \Phi + \Phi \frac{(\rho c_p)_s}{(\rho c_p)_f} \right)$$

where the subscripts f denote base fluid while s represent the nanoparticle. In this study Φ represent nanoparticle volume fraction. The thermal properties of base fluid and nanoparticle are in the table below.

Table1: Thermal physical properties of pure water, copper and silver nanoparticles

Physical properties	Base fluid (water)	Copper	Silver (Ag)
c_p(j/kgk)	4179	385	235
$\rho(kg/m^3)$	997.1	8933	10500
$k(w/mk)$	0.613	401	429

$\alpha \times 10^7 (m^2/s)$	1.47	1163.1	1738.6
$\beta \times 10^5 (k^{-1})$	21	1.67	1.89

Governing equations

Velocity, temperature and concentration in the boundary layer are governed by continuity, momentum, thermal energy and concentration equations listed below.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0. \tag{1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho_{nf}}\frac{\partial p}{\partial x} + \vartheta_{nf}\frac{\partial^2 u}{\partial y^2} + g_a B_T (T - T_\infty) + g_a B_C (C - C_\infty) - \sigma B_0^2 u. \tag{2}$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha_{nf}\frac{\partial^2 T}{\partial y^2} + \frac{\rho_f D_m k_T}{C_s \rho_{nf}(c_p)_{nf}}\frac{\partial^2 C}{\partial y^2} + \frac{Q}{\rho_{nf}(c_p)_{nf}}(T - T_\infty). \tag{3}$$

$$\frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D_m\frac{\partial^2 C}{\partial y^2} + \frac{D_m k_m}{T_m}\frac{\partial^2 T}{\partial y^2} - R(C - C_\infty). \tag{4}$$

Where u, v, T and C denote velocity component along x axis, velocity component along y axis, temperature and concentration respectively. In the free stream momentum (2) reduces to

$$\frac{1}{\rho_{nf}}\frac{\partial p}{\partial x} = -U_\infty\frac{dU_\infty}{dx} - \sigma\frac{B_0^2}{\rho_{nf}}U_\infty \tag{5}$$

Substituting equation (5) into equation (2) momentum equation simplifies to

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \vartheta_{nf}\frac{\partial^2 u}{\partial y^2} + U_\infty\frac{dU_\infty}{dx} + (U_\infty - u)\sigma\frac{B_0^2}{\rho_{nf}} + g\beta_T(T - T_\infty) + g_a\beta_C(C - C_\infty). \tag{6}$$

In order to satisfy continuity equation a stream function given by Haroun *et al* (2015)

$$\varphi = \sqrt{b_\infty\vartheta_f\varepsilon}\, xf(\varepsilon, n), \tag{7}$$

is introduced and

$$u = \frac{\partial\varphi}{\partial y}, v = -\frac{\partial\varphi}{\partial x}. \tag{8}$$

Non-dimensionalizing governing equations

Equations (1, 3, 4 and 6) are nondimensionalized using dimensionless variables; namely dimensionless length along y direction n, dimensionless time ε, dimensionless stream function $f(\varepsilon, n)$, dimensionless temperature $\theta(\varepsilon, n)$ and dimensionless concentration

$$y = n\sqrt{\frac{\vartheta_f \varepsilon}{b_\infty}}, \quad \varepsilon = 1 - e^{b_\infty t}, \quad f(\varepsilon, n) = \frac{1}{\sqrt{a_\infty \vartheta_f \varepsilon x}}, \quad \frac{\theta(n)}{g(\varepsilon)} = \frac{T - T_\infty}{T_w - T_\infty}$$

$$\frac{\emptyset(n)}{g(\varepsilon)} = \frac{C - C_\infty}{C_w - C_\infty}$$

(9)

Using equations (8) and (10) the governing equations (3), (4) and (6) reduces to

$$f''' + \Phi_0 \left[(1 - \varepsilon) \frac{n}{2} f'' + \varepsilon \left(-f'^2 + ff'' + 1 + Ha^2(1 - f') + Gr_T \frac{\theta}{g(\varepsilon)} + Gr_C \frac{\emptyset}{g(\varepsilon)} \right) \right] = \Phi_0 \varepsilon (1 - \varepsilon) \frac{\partial}{\partial \varepsilon} f'$$

(10)

$$\theta'' + \frac{k_f}{k_{nf}} pr \Phi_a \left\{ \frac{n}{2} (1 - \varepsilon) \theta' + \varepsilon [f\theta' + \sigma\theta] + \frac{D_f}{\Phi_a} \emptyset'' \right\} = -\frac{k_f}{k_{nf}} pr \Phi_a \varepsilon (1 - \varepsilon) \frac{\theta g_\varepsilon}{g}.$$

(11)

$$\emptyset'' + Sc \left[\frac{n}{2} (1 - \varepsilon) \emptyset' + \varepsilon (f\emptyset' - \gamma\emptyset) + Sr\theta'' \right] = -Sc\varepsilon (1 - \varepsilon) \frac{\emptyset}{g} g_\varepsilon.$$

(12)

The non-dimensionalized boundary conditions and initial conditions are respectively

$$f(\varepsilon, 0) = f_w, f'(\varepsilon, 0) = \lambda, \theta(\varepsilon, 0) = \varepsilon, \emptyset(\varepsilon, 0) = \varepsilon \text{ at } n = 0, \varepsilon \geq 0$$

(13)

$$f'(\varepsilon, \infty) = 1, \theta(\varepsilon, \infty) = 0, \emptyset(\varepsilon, \infty) = 0, \text{as } n \to \infty, \varepsilon \geq 0$$

(14)

Equations (11), (12) and (13) can be simplified further by assuming initially the flow is steady and unsteadiness parameter, $g(\varepsilon) = \varepsilon$. This results to

$$\frac{d^3 f}{dn^3} + \Phi_0 \frac{n}{2} \frac{d^2 f}{dn^2} + \Phi_0 [Gr_T \theta + Gr_C \emptyset] = 0.$$

(15)

$$\frac{d^2 \theta}{dn^2} + \frac{1}{2} \frac{k_f}{k_{nf}} Pr \Phi_a n \frac{d\theta}{dn} + \frac{k_f}{k_{nf}} Pr D_f \frac{d^2 \emptyset}{dn^2} = -\Phi_a \frac{k_f}{k_{nf}} Pr \theta.$$

(16)

$$\frac{d^2 \emptyset}{dn^2} + \frac{1}{2} Scn \frac{d\emptyset}{dn} + ScSr \frac{d^2 \theta}{dn^2} = -Sc\emptyset.$$

(17)

where Φ_0 *and* Φ_a are constants.

From the non dimensionalizing the following dimensionless parameters were obtained:

$$Pr = \frac{\vartheta_{nf}}{\alpha_{nf}}, Gr_T = \frac{g\beta_T(T_w - T_\infty)}{b_\infty^2 x}, Gr_c = \frac{D_m K_T(C_w - C_\infty)}{b_\infty^2 x}, Ha^2 = \frac{\sigma B_0^2}{b_\infty \rho_{nf}}, \sigma = \frac{Q}{b_\infty (\rho c_p)_{nf}}, D_f$$

$$= \frac{D_m k_T(C_w - C_\infty)}{C_s(c_p)_f \vartheta_f(T_w - T_\infty)}, sc = \frac{\vartheta_{nf}}{D_m}, \gamma = \frac{R}{b_\infty}, sr = \frac{D_m k_T(T_w - T_\infty)}{T_m \vartheta_f(C_w - C_\infty)}$$

Equations (15), (16) and (17) are discretized using (FVM). FVM employs integration over a control volume. FVM is the best because it is stable, consistent, convergent, conservative and bound. The discretized equation are obtained using the computational grid below.
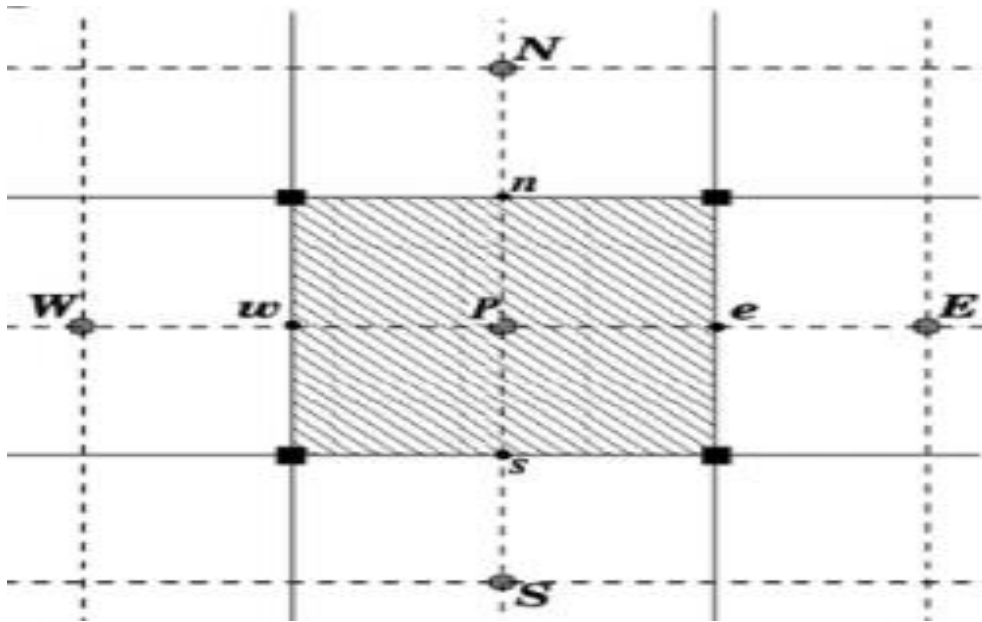

Figure 1: Computational grid.

FVM linearize and decoupled system of differential equations (15),(16) and (17) into

$$\left[\frac{1}{\Delta n} - \frac{1}{8}\Phi_0(n_i + n_{i-1})\right]f'_{i-1} + \left[\frac{-2}{\Delta n} + \frac{1}{8}\Phi_0(n_{i+1} - n_{i-1}) - \frac{1}{2}\Phi_0\Delta n\right]f'_i + \left[\frac{1}{\Delta n} + \frac{1}{8}\Phi_0(n_i + n_{i+1})\right]f'_{i+1} = -\Phi_0[Gr_T\theta_i + Gr_C\emptyset_i]\Delta n$$

(18)

$$\left(\frac{1}{\Delta n} - \frac{1}{8}\frac{k_f}{k_{nf}}pr\Phi_a(n_i + n_{i-1})\right)\theta_{i-1} + \left(\frac{-2}{\Delta n} + \frac{1}{8}\frac{k_f}{k_{nf}}pr\Phi_a(-3n_{i+1} - n_{i-1} + 4n_i) + 8\right)\theta_i + \left(\frac{1}{\Delta n} + \frac{1}{8}\frac{k_f}{k_{nf}}pr\Phi_a(n_{i+1} + n_i)\right)\theta_{i+1} = -\frac{k_f}{k_{nf}}prD_f\left(\frac{\emptyset_{i+1}}{\Delta n} - \frac{2\emptyset_i}{\Delta n} + \frac{\emptyset_{i-1}}{\Delta n}\right)$$

(19)

$$\left(\frac{1}{\Delta n} - \frac{1}{8}Sc(n_i + n_{i-1})\right)\emptyset_{i-1} + \left(\frac{-2}{\Delta n} + \frac{1}{8}Sc(-3n_{i+1} - n_{i-1} + 4n_i + 8)\right)\emptyset_i + \left(\frac{1}{\Delta n} + \frac{1}{8}Sc(n_{i+1} + n_i)\right)\emptyset_{i+1} = -\frac{ScSr}{\Delta n}(\theta_{i+1} - 2\theta_i + \theta_{i-1})$$

(20)

RESULTS AND DISCUSSION

In this study non dimensioned and discretized the governing equations have been done. Also to comprehend the flow problem physically numerical computations are done by varying the volume fraction, Φ and results are illustrated graphically. The variation is to determine the effect of volume fraction to velocity, temperature and concentration profiles. Copper and silver are used as the nanoparticle and water as base fluid. The graphs were plotted using Table1, $\Phi = 0.2, 0.4, 0.6, Df = 0.01, \lambda = 0.5, Gr_T = 0.01, Gr_C = 0.01, Pr = 7, Ha = 2, \delta = 0.1, sc = 1, sr = 1,, \gamma = 0.1 \text{ and } \varepsilon = 0.$ From the graphs it is true that velocity decrease when nanoparticle volume fraction is increased, temperature increase with increase of nanoparticle volume fraction, while the concentration decrease.
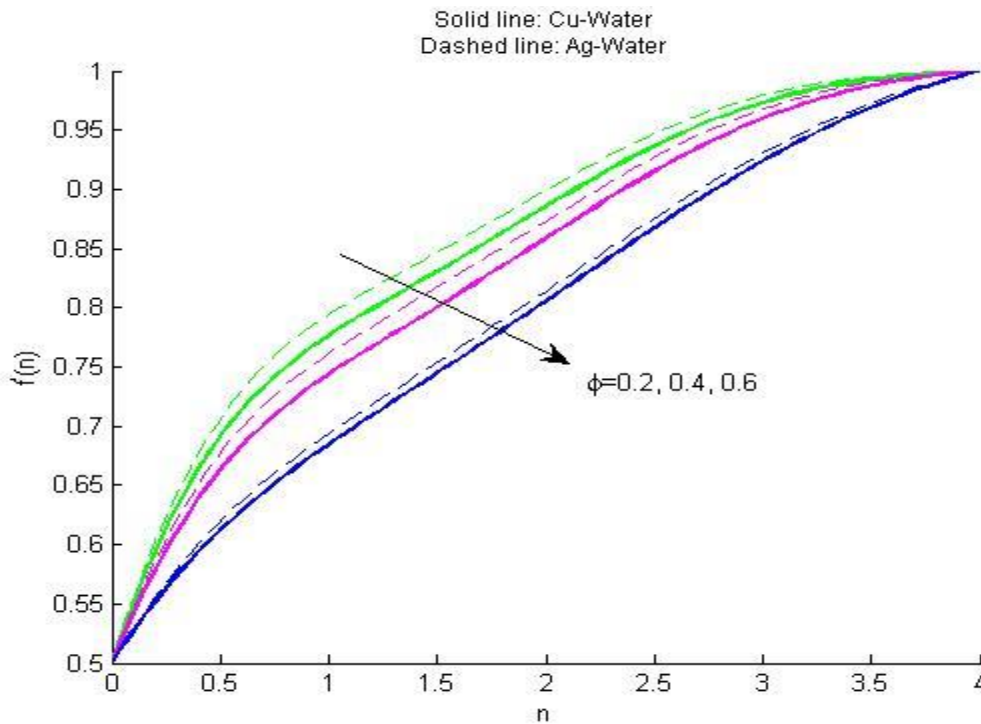

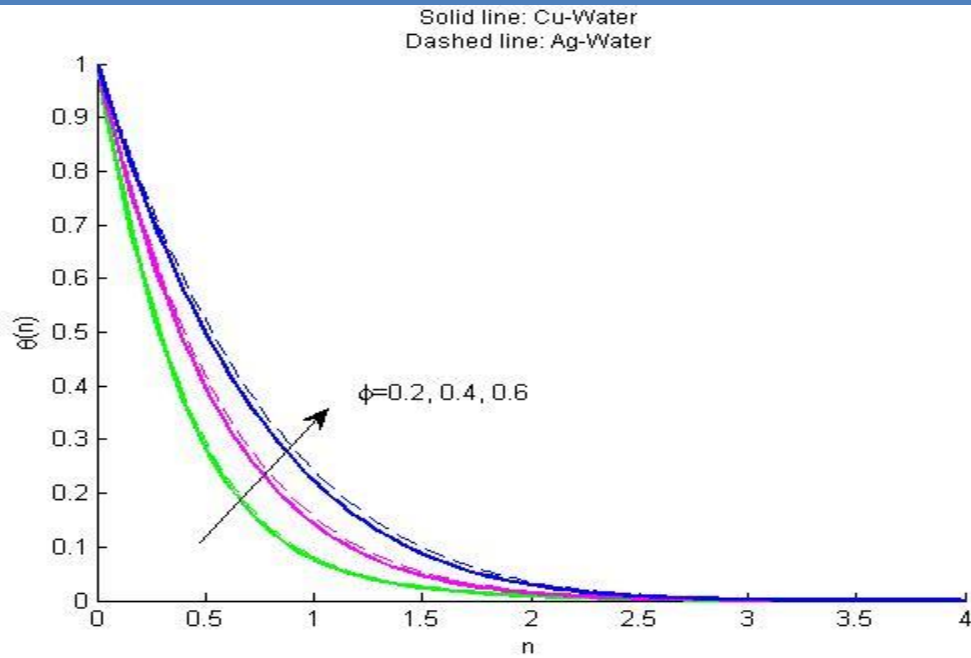**Figure 1: Effect of variation of nanoparticle volume fraction on velocity**

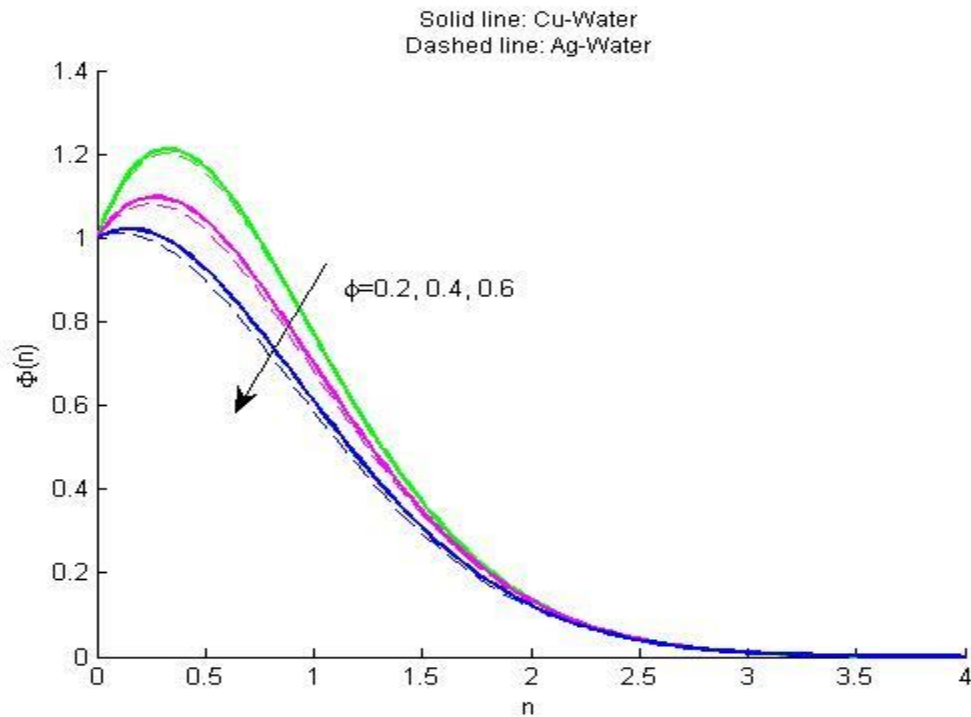**Figure2: Effect of variation of nanoparticle volume fraction on temperature**

Figure 2: Effect of variation of nanoparticle volume fraction on concentration

CONCLUSION

Non dimensiolization of equation governing the flow has been achieved. FVM algorithm for solving the governing equation has been developed. FVM has transformed the coupled momentum, energy and concentration equations to system of linear equations. Nanoparticle volume fraction significantly affect nanofluid velocity, temperature and concentration.

Abbreviation

b: Positive constant

v_w: suction velocity

T_w: Surface temperature

C_w: Surface concentration

T_∞: Ambient temperature

C_∞: Ambient concentration

u: Fluid velocity component along x direction

v: Fluid velocity component along y

ρ_{nf}: Nanofluid   density

p: Fluid pressure

ϑ_{nf}: Nanofluid kinematic viscosity

g: Unsteadiness parameter

g_a Gravitational acceleration

B_C: Volumetric solutal expansion coefficient

σ: Electrical conductivity

B_0: Magnetic field

T: Fluid temperature

α_{nf}: Nanofluid thermal diffusivity

ρ_f: Fluid density

D_m: Concentration mass diffusivity

C_s: Concentration susceptibility

$(c_p)_{nf}$: Nanofluid specific heat capacity at constant pressure

C: Fluid concentration

Q: Volumetric rate of heat generation

k_m: mean fluid

R: Chemical reaction parameter

T_m: mean fluid.

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