



MACHAKOS UNIVERSITY

University Examinations for 2020/2021 Academic Year

SCHOOL OF BUSINESS AND ECONOMICS

DEPARTMENT OF ECONOMICS

SECOND YEAR SPECIAL/SUPPLEMENTARY EXAMINATION FOR

BACHELOR OF ECONOMICS AND STATISTICS

BACHELOR OF ECONOMICS AND FINANCE

BACHELOR OF ECONOMICS

BACHELOR OF ARTS

EES 200: MATHEMATICS FOR ECONOMICS II

DATE: 26/3/2021

TIME: 8.30-10.30 AM

INSTRUCTIONS:

- (i) Answer question one (Compulsory) and any other two questions
- (ii) Do not write on the question paper
- (iii) Show your workings clearly

QUESTION ONE (30 MARKS)

a) Differentiate the following functions

i. $y = 5x^3 + \frac{2}{e^{x^2}}$ (2 marks)

ii. $y = \ln(5x^2 - 2)$ (2 marks)

b) The following input – output model shows the relationship between two industries; agriculture and manufacturing sectors in the economy for the year 2019.

	Agriculture	Manufacturing	Final demand	Total
Agriculture	100	50	360	510
Manufacturing	80	200	100	380

Required:

- i. Calculate the input – output coefficients in this 2-sector industry (4 marks)
- ii. If the final demand in the year 2013 is given by: $\begin{bmatrix} 500 \\ 200 \end{bmatrix}$

- Calculate the level of output required in each of the 2 sectors of the economy (6 marks)
- c) Find the product of the following two matrices (5 marks)
- $$A = \begin{bmatrix} 8 & 1 & 3 \\ 4 & 0 & 1 \\ 6 & 0 & 3 \end{bmatrix} \quad B = \begin{bmatrix} 10 & 3 \\ 7 & -1 \\ -6 & -7 \end{bmatrix}$$
- d) Verify Euler's theorem for the function, $Q = L^3 + K^3 + L^2K$. (5 marks)
- e) A monopolist producing two goods Q_1 and Q_2 has the following demand functions corresponding to each of the two goods: $P_1 = 56 - 4Q_1$ and $P_2 = 36 - 3Q_2$. Find the prices and output that maximizes total revenue for the firm (6 marks)

QUESTION TWO (20 MARKS)

- a) Find $\frac{\partial y}{\partial x_1}$ and $\frac{\partial y}{\partial x_2}$ for the following functions and state the rule used
- i) $y = (4x_1^3 + x_1x_2)(x_1^2 + 3)^3$ (3 marks)
- ii) $y = \frac{6x_1 - x_2^3}{x_2^3}$ (3 marks)
- b) The following information relates to national income model:

$$Y = C + I \quad C = 60 + 0.8Y \quad I = 50$$

Present this information in matrix algebra and then use matrix inversion method to solve for \bar{C} (5 marks)

- c) A multiproduct firm is faced with the following cost function and a production constraint. The production constraint is stipulated in terms of a production quota.

$$\text{Cost function: } C = 2Q_1^2 + 4Q_2^2 - Q_1Q_2 + 10$$

$$\text{Production quota: } Q_1 + Q_2 = 16$$

- i. Set up a constrained cost minimization problem and then construct the Lagrangian function (4 marks)
- ii. Solve for the optimal values of output (5 marks)

QUESTION THREE (20 MARKS)

- a) For some firm, the number of units produced when using x units of labour and y units of capital is given by the production function $f(x, y) = 80x^{0.25}y^{0.75}$
- i. Find the equations for both marginal products (3 marks)
- ii. What is the nature of the marginal products? (3 marks)

- b) Given $U = (x+2)(y+1)$ and $P_x=4$, $P_y=6$ and $M = 130$. Write the langragian function and then find the optimal levels x , y and λ (8 marks)
- c) The demand function for a product is $P = 81 - x^2$ and the supply function is $P = x^2 + 4x + 11$. Find the consumer's and producer's surplus (6 marks)

QUESTION FOUR (20 MARKS)

- a) Given the function $Q = 2k^2 - 3kL + L^2$, determine the nature of returns to scale for the function and interpret (4 marks)
- b) Find the derivative of the following function: $y = f(x) = x^2 e^x$ (4 marks)
- c) For the following two demand functions, compute the partial elasticities of demand and state whether the commodities are substitutes or complements
- $$Q_1 = 7 - 2P_1 - P_2$$
- $$Q_2 = 23 - P_1 - 3P_2$$
- (8 marks)
- d) If $Z = f(x, y) = (x^2 + 4x - 5y^3)^5$ find $\frac{\delta Z}{\delta x}$ and $\frac{\delta Z}{\delta y}$ (4 marks)

QUESTION FIVE (20 MARKS)

- a) Prove Young's Theorem using the following function
- $$f(x_1, x_2, x_3) = 2x_1^3 + 5x_1x_2^2 - x_3^4$$
- (4 marks)
- b) Evaluate the following $\int_3^6 \frac{x}{3x^2 + 4} dx$ (3 marks)
- c) Find the minimum value of the function $Z = x^3 + y^3 + xy$ subject to the constraint $x + y - 4 = 0$ (6 marks)
- d) The demand and supply of two products are given by the functions
- $$Q_{d1} = 10 - 2P_1 + P_2$$
- $$Q_{s1} = -2 + 3P_1$$
- $$Q_{d2} = 15 + P_1 - P_2$$
- $$Q_{s2} = -1 + 2P_2$$
- Use cramer's rule to solve for the equilibrium prices and quantities (7 marks)