MACHAKOS UNIVERSITY

University Examinations for 2020/2021 Academic Year

SCHOOL OF BUSINESS AND ECONOMICS

DEPARTMENT OF ECONOMICS

THIRD YEAR SPECIAL/SUPPLEMENTARY EXAMINATION FOR

BACHELOR OF ECONOMICS AND STATISTICS

BACHELOR OF ECONOMICS AND FINANCE

BACHELOR OF ECONOMICS

BACHELOR OF ARTS

EES 300: MATHEMATICS FOR ECONOMISTS III

DATE: 26/3/2021   **TIME: 11.00-1.00 PM**

INSTRUCTIONS:

        Answer Question ONE and any other TWO questions

QUESTION ONE (COMPULSORY) (30 MARKS)

a)   The marginal costs of two firms are given by the following functions:

         1) $C'(Q) = 15 + 20Q - 12Q^2$ $TC = 100$ when $Q = 0$

         2). $C'(Q) = 100e^{0.2Q}$ $TC = 3000$ when $Q = 0$

       i.   Find the total cost function C(Q) for each firm.   (4 marks)

       ii.   Find the average cost function for each firm   (4 marks)

       iii.   What is the fixed costs of each firm   (2 marks)

b)   Find the time path of capital K(t) given the following rates of net investment flow functions

       i. $I(t) = 5t^{1/2}$ $K(0) = 500$ (3 marks)

       ii. $I(t) = 18t^{3/5}$ $K(0) = 30$ (3 marks)

       iii.   For each of (i) to (ii) above, find the amount of capital formation over the interval [0,5]   (6 marks)

c)   Find the producer surplus of the following

$$Q = \sqrt{P - 6} \text{ given that } \overline{P} = 20$$ (4 marks)

d)   Verify that the following differential equation is exact and solve the equations

$$8ytdy + (4y^2 + 3t)dt = 0$$ (4 marks)

QUESTION TWO (20 MARKS)

a)   Derive general solution of the First order differential equations (FOLDE)   (6 marks)

b)   Suppose you are given the following demand and supply functions

$$Qd = \alpha - \beta P \qquad\qquad (\alpha, \beta > 0)$$

$$Qs = -\gamma + \delta P \qquad\qquad (\gamma, \delta > 0)$$

   i.   Assuming that the rate of change of price over time is directly proportional to the excess demand, find the time path P(t) (general solution   (6 marks)

   ii.   What is the inter-temporal equilibrium price   (2 marks)

   iii.   What is the market clearing equilibrium price   (2 marks)

   iv.   Does the market have a dynamically stable equilibrium price? Explain   (4 marks)

QUESTION THREE (20 MARKS)

a)   Find the general and definite solution to the following differential equations

   i) $\dfrac{dy}{dt} + 4y = 10 \qquad\qquad y(0) = 12$ (3 marks)

   ii) $\dfrac{dy}{dt} + 10y = 12 \qquad\qquad y(0) = 20$ (3 marks)

b)   Find the integral of the following

   i) $\int \left(8x^2 e^{(X^3 + 10)} + \dfrac{4}{x^3} \right) dx \qquad\qquad (x \neq 0)$ (3 marks)

   ii) $\int (1n\ x)^3 dx$ (3 marks)

c)   Solve the following equations using matrix algebra   (8 marks)

$$2x + y + 3w = 15$$

$$x + 3y + w = 10$$

$$3x + 2y + 2w = 20$$

QUESTION FOUR (20 MARKS)

a)   Solve the following difference equations

   i. $y_{t+1} = y_t + 4$ ($y_0 = 8$)   (3 marks)

   ii. $y_t = 3y_{t-1} + 4$ ($y_0 = 1$)   (3 marks)

b)   For the general first difference equation given as follows:

   $$y_{t+1} + \alpha y_t = \beta$$

   i.   Find the general solution in the case where $(a \neq -1)$ (8 marks)

   ii. Decompose the general solution into two components, the complementary function y_c and the particular integral y_p and interpret each term.   (4 marks)

   iii. Which of the two components in (ii) above determines whether the equilibrium is dynamically stable or not   (2 marks)

QUESTION FIVE (20 MARKS)

a)   Find Y and r by Cramer's rule and inverse matrix, given the following IS-LM models

   (10 marks)

   Goods Market   Money Market

   $Y = C + I$ $Md = 100 + 0.5Y + 0.25r$

   $Y = 200 + 0.2Y$ $Ms = 120$

   $I = 8 - 0.3r$ $Md = Ms$

b)   Find the consumer surplus given the following demand function   (6 marks)

   $Q_d = 25 - \frac{1}{3}P$ and Quantity supply given as $Q_s = -1 + \frac{1}{2}P$

c)   The growing value of GNP is given by:

   $$GNP_t = GNP_0 e^{rt} \qquad\qquad r = 1.5\%$$

   i.   If $GNP_0 = 500$, find the value of GNP 10 years from now   (2 marks)

   ii. If $GNP_0 = 1000$, after how many years will the GNP double?   (2 marks)