

### DATE: 21/1/2021

TIME: 2.00-4.00 PM

## **INSTRUCTIONS:**

- (i) Answer question one (Compulsory) and any other two questions
- (ii) Do not write on the question paper
- (iii) Show your workings clearly

#### **QUESTION ONE (COMPULSORY) (30 MARKS)**

a)	Defi	Define the following as used in finance				
	i.	Law of one price	(2 marks)			
	ii.	Bid-Ask spread	(2 marks)			
	iii.	Arbitrage and strong arbitrage	(2 marks)			

- iv. Risk free asset (2 marks)
- v. Redundant Security (2 marks)
- b) Mercy, a K24 student at Machakos University cannot figure out why market completeness is critical in a one period framework security market. Explain to her market completeness is as well as its implications. (7 marks)
- c) Consider a financial market with a money account, a stock, and a European call option on the stock with strike price of KES196. Suppose there are two future states. If state 1 realizes, the stock price declines to KES 168 from the current price of KES200. If state 2 happens, the stock price rises to KES224. Suppose the interest rate on the money account is 5% no matter

the state and assuming that the payoffs on the stock and the money account are the stock prices and compounded money value respectively in the different date 1 states

i. Complete the table below for the payoffs and clearly state the rationale for your answers (2 marks)

	Date 0	Date 1 (Payoffs)	
	Price	State 1	State 2
Money Account	1		
Stock	200		
Call Option	C <sub>0</sub>		

- ii. At what price should we price the call option to avoid arbitrage (3 marks)
- iii. Would there be an arbitrage if the price were 15 or 25 (8 marks)

#### **QUESTION TWO (20 MARKS)**

A security market has three securities. Security one has a payoff of 10 in state one, -5 in state two and a payoff of 3 in state 3. Security two has a payoff of -7 in state one, 14 in state two and a payoff of -5 in state 3. Security three has payoff of -1 in state one, -5 in state two and a payoff of 7 in state 3. If the security prices are 55, 27 and 18 for security one, two and three respectively;

- a) Form the payoff matrix for this market (1 mark)
- b) Find and interpret the state prices given that  $p_j = \sum_s x_{j_s} \frac{u_1'(c_0, c_1)}{u_0'(c_0, c_1)}$  (value of a security= sum
  - [payoffs in future states X state prices]). (10 marks)
- c) Find and interpret the risk-free rate in this market (1 mark)
- d) Find the price of a put option placed on the 3<sup>rd</sup> security if its strike price KES 5 given that the payoffs in the payoff matrix represent the future prices of the securities in the various states in date 1 (3 marks)
- e) what is the replicating portfolio for this put option? Does the replicating portfolio have the same price as the put option given the date zero prices of security 1, 2 and 3 (5 marks)

## **QUESTION THREE (20 MARKS)**

- a) Given that y = mx + c where m and c are constants, use the properties of mean and variance to prove that;
  - i. E(y) = c + mE(x) Where E is the expectations operator (1 mark)
  - ii.  $Var(y) = m^2 Var(x)$  (2 marks)

b) Given that securities X and Y are perfectly correlated such that

probability	$x_i \%$
0.2	11
0.2	9
0.2	25
0.2	7
0.2	-2

Y = 11.15789 - 0.315789X and the probability distribution of X is;

Find; E(X) and Var(X) (3 marks)

E(Y) and Var(Y)

(3 marks)

- c) Write down the equations for expected portfolio return and variance for a portfolio consisting of security X and Y if *a* % is invested in security X and *b* % is invested in Y (2 marks)
- d) Use the equations in (c) above to complete the following table given that cov(x, y) = -0.0024.  $E(R_p)$  and  $\sigma(R)$  are expected portfolio returns and standard deviation of portfolio returns respectively. (6 marks)

Percentage in X	percentage in Y	$E(R_p)\%$	$\sigma(R) = \sqrt{\left(\operatorname{var}(R_p)\right)}$	$\sigma\%$
100	0			
75	25			
50	50			
25	75			
0	100			

e) Use the table in (d) above to demonstrate the significance of portfolio diversification

(3 marks)

# **QUESTION FOUR (20 MARKS)**

a) Explain in detail any three interlinkages between financial markets and the macroeconomy

(9 marks)

b) State the agents consumption portfolio choice problem under short sale restrictions

(1 mark)

- c) Let there be two securities: a risky security with return denoted by r and a risk free security with return  $r^*$ . For a portfolio (h<sub>1</sub>, h<sub>2</sub>) such that  $p_1h_1 + p_2h_2 = w$ , let  $a = p_2h_2$  denote the amount invested in the risky security. Required;
  - i. Represent portfolio  $(h_1, h_2)$  in terms of wealth (3 marks)
  - ii. What significance does the reformulation in (i) above have? (2 marks)
  - iii. State the payoff of the portfolio in (i) above in terms of w, a, r, and  $r^*$  (2 marks)
  - iv. State the agents portfolio choice problem given that the representative agents utility function is u = V(z), where z is a portfolio payoff. (3 marks)

## **QUESTION FIVE (20 MARKS)**

Consider the following three securities which give the stated payoffs at the stated probabilities

Secu	rity 1	security 2		Security 3	
Payoff	Prob.	Payoff	Prob.	Pay off	Prob.
4	0.25	1	0.33	6	0.20
5	0.50	6	0.33	10	0.70
12	0.25	8	0.33	13	0.10

a) Draw the pairwise cumulative distribution curves for the payoffs in separate panels

(12 marks)

- b) which pairs of these securities exhibit first order stochastic dominance and second order stochastic dominance (4 marks)
- c) Is it true to say that: (2 marks) payoffs of security 3 = payoffs of security 1+'something good'
- d) If true/not true why? (2 marks)