MACHAKOS UNIVERSITY

University Examinations for 2019/2020 Academic Year

SCHOOL OF BUSINESS AND ECONOMICS

DEPARTMENT OF ECONOMICS

SECOND YEAR SPECIAL/SUPPLEMENTARY EXAMINATION FOR

BACHELOR OF ECONOMICS AND STATISTICS

BACHELOR OF ECONOMICS

EES 200: MATHEMATICS FOR ECONOMICS II

DATE: 22/1/2021 **TIME: 8.30-10.30 AM**

INSTRUCTIONS:

Answer Question ONE and any other TWO questions

QUESTION ONE (30 MARKS)

a) Give the following national income identity for open economy: (5 marks)

$$Y = C + I_0 + G_0 + X_0 - M$$

$$C = c_0 + c_1(Y^d)$$

$$T = t_0 + t_1 Y$$

$$M = m_0 + m_1 Y$$

$$Y^d = Y - T$$

Y, C and T are endogenous variables while I_0 and G_0 are exogenous variables.

Find equilibrium Y, C and T

b) Given the following demand and supply and equilibrium functions (7 marks)

$$Q_d = a - bP$$

$$Q_s = -c + dP$$

$$Q_d = Q_s$$

Find equilibrium p and q and evaluate how the two changes with parameter

c) A system of two simultaneous equations is given as: (7 marks)

$X + 2Y = 10$

$5X + 8Y = 40$

Determine the values of X and Y using the Cramer's rule

d) Explain different types of return to scale. (5 marks)

e) Find determinants of the following: (6 marks)

i) $\begin{bmatrix} 8 & 0 \\ 4 & 1 \end{bmatrix}$

ii) $\begin{bmatrix} 4 & 2 & 0 \\ 8 & 9 & 2 \\ 1 & 5 & 3 \end{bmatrix}$

QUESTION TWO (20 MARKS)

a) Partial derivatives of utility function yields marginal utility whereby second derivative determines the nature of Marginal utility. Explain why marginal utility can be positive and to some level negative (5 marks)

b) Find the total differential of the following utility functions (6 marks)

i) $Y = 4X_1 + 10X_2 X_1^2 + 3X_2^3$

ii) $Z = 5XY + 2X^2Y - 8Y^2X$

iii) $q = \dfrac{x^{\alpha} l^{\beta}}{x^{\phi}}$

c) Assume that a demand function is given as: (5 marks)

$P = 40 - 2Q$

Determine the consumer surplus when $market\ price = 10$

d) Given the production function below proof the Euler's theorem algebraically. (5 marks)

$Q = 3K^3 - 5KL^2 - L^3$

e) Find the integral of the following: (4 marks)

i. $\int Ax^n . dx$

ii. $\int e^x . dx$

iii. $\int \frac{1}{x} dx$

iv. $\int dx$

QUESTION THREE (20 MARKS)

a) A production function is given as: (8 marks)

$Q = 80K^{0.2}L^{0.5}$

Determine the marginal product of labour, marginal product of capital, the slope of the production function and determine the return to scale of the function

b) Describe the relationship between the homogeneity of a production function and return to scale of a production function. (2 marks)

c) Determine the degree of homogeneity of the function given as: (4 marks)

$Y = X^2 + 5XW + W^2$

d) Get the derivative of Y with respect to t for the equation below (6 marks)

$y = t^3\ Lnt^2$

QUESTION FOUR (20 MARKS)

a) Determine the nature of returns to scale for the following function: (6 marks)

i) $Q = 80K^{0.2}L^{0.5}$

ii) $Q = A\,L^{\frac{1}{2}}K^{\frac{3}{4}}$

b) A demand function is given as: (9 marks)

$Q_A = 90 - 2P_A + 0.4P_B + 0.1Y$

Find the price, income and cross price elasticities of demand if:

$P_A = 6 \quad P_B = 10 \quad Y = 1000$

c) Describe Euler's theorem (2 marks)

d) Given the production function below proof the Euler's theorem algebraically. (3 marks)

$Q = AK^{\alpha}L^{\beta}$

$Q = 3K^3 - 5KL^2 - L^3$

QUESTION FIVE (20 MARKS)

a) Assume the following equation is a maximization problem. (6 marks)

$$Z = x^2 + y^2 + xy$$

Subject to : $x + 3y = 13$

Require : Solve for x, y and λ

b) Given a utility function U=5xy, and a budget constraint given as 5x+y=30, determine the levels of x and y that will maximize the utility of the consumer. (4 marks)

c) The demand supply functions for a commodity are given as: (5 marks)

$Q_d = 6 - P^2$ Demand equation

$Q_s = 10P - 5$ Supply equation

Determine equilibrium price and quantity

d) The demand and supply functions of 2 commodities are given as:

$Q_{d1} = 20 - P_1 + P_2$ $Q_{s1} = 16 - P_1 + 3P_2$

$Q_{d2} = 36 - 5P_1 + 4P_2$ $Q_{s2} = 10 - P_1 + 9P_2$

Determine the equilibrium prices and quantities for the 2 goods. (5 marks)