



# MACHAKOS UNIVERSITY

University Examinations for 2020/2021 Academic Year

SCHOOL OF BUSINESS AND ECONOMICS

DEPARTMENT OF ECONOMICS

FIRST YEAR SPECIAL/SUPPLEMENTARY EXAMINATION FOR

BACHELOR OF ECONOMICS AND STATISTICS

BACHELOR OF ECONOMICS AND FINANCE

BACHELOR OF ECONOMICS

EES 100: MATHEMATICS FOR ECONOMISTS 1

DATE: 26/3/2021

TIME: 8.30-10.30 AM

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## INSTRUCTIONS:

- (i) Answer question one (Compulsory) and any other two questions
- (ii) Do not write on the question paper
- (iii) Show your working clearly

## QUESTION ONE (COMPULSORY) (30 MARKS)

The economy of a given country depends on three main sectors namely; Agriculture (A), Tourism (T) and Industry (I). Agriculture consists of two main elements namely; Livestock production (L) and Crop production (C). Tourism consists of three main elements namely; Wildlife (W), Scenery (S) and Recreation (R). Industry consists of two main elements namely; Motors (M) and Physical environment (P) whose contribution to the economy is the same as that of Wildlife (W), implying that  $W = P$ .

- a) Given that the associative law is given as  $A \cup (T \cap I) = (A \cup T) \cap I$  and  $A \cap (T \cup I) = (A \cap T) \cup I$ , represent the above information in form of three main sets, that is, A, T and I. Also verify the associative law (7 marks)
- b) Suppose that the demand function is linear, say  $q = D(p) = -400p + 1400$ , where p is measured in dollars and q in hot-dogs sold per day. Also suppose that the cost of each hot-dog is \$0.50 and the daily fixed costs are \$500.
  - i. Determine the revenue function (1 mark)

- ii. Determine the value of p, q and revenue (3 marks)
  - iii. Determine the cost function in terms of p (1 mark)
  - iv. Determine the profit maximizing price, demand, revenue and cost. Also determine the value for maximum profit (5 marks)
- c) The commodity and money markets for an economy are defined by the following equations:

Commodity Market

$$Y = C + I$$

$$C = 200 + \frac{2}{5}Y$$

$$I = 1900 - 12r$$

Money Market

$$M_{DT} = \frac{1}{2}Y$$

$$M_{DS} = 100 - 10r$$

$$M_S = 1500$$

- (i) Derive the IS and the LM functions for the economy (4 marks)
  - (ii) Determine the equilibrium income and the rate of interest for the economy (5 marks)
- d) The supply and demand functions for a firm are given by

$$Q_d = 60 - \frac{1}{3}P$$

$$Q_s = -30 + \frac{2}{3}P$$

If the government decides to impose a per unit tax t on quantity supplied, find the tax rate that will maximize government tax revenue (4 marks)

### QUESTION TWO (20 MARKS)

- a) An economy is described by the following production function:

$$Q = 2K^2 - 3L^2$$

Demonstrate Euler's theorem

(3 marks)

- b) You are given the following information about a firm

$$2 - \frac{4}{Q} = Q - ATC$$

$$4Q = 2 - AR$$

- i. Find the price elasticity of demand at P = 4 (3 marks)

- ii. At what level of Q is ATC at a minimum? (3 marks)
- iii. Find the profit maximizing Q if a tax of 2 per unit is imposed on quantity produced (2 marks)
- iv. Find the profit maximizing Q if a subsidy of 2 per unit is imposed on quantity produced (2 marks)
- c) The demand and average total costs facing a firm are given below:
- $$P = 6 - \frac{3}{5}Q$$
- $$ATC = Q - \frac{2}{5} + \frac{3}{Q}$$
- i. Find Q when profits equal to zero (4 marks)
- ii. Determine the level of Q that corresponds to a profit level of -3 (3 marks)

### QUESTION THREE (20 MARKS)

- a) Consider the following demand function
- $$Q = \frac{1}{P^k}$$
- i. Determine  $\frac{\epsilon_d}{p}$ , the price elasticity of demand for the function. (4 marks)
- ii. Show that the relation  $MR = P(1 + \frac{1}{\epsilon_d/P})$  between marginal revenue and elasticity of demand is valid. (4 marks)
- b) Consider the following production function and the corresponding marginal products
- $$Q = A X_1^{\alpha_1} X_2^{\alpha_2} X_3^{\alpha_3} \quad \alpha_1 + \alpha_2 + \alpha_3 = 1$$
- i. Determine  $MPX_1$  (1 mark)
- ii. Determine  $MPX_2$  (1 mark)
- iii. Determine  $MPX_3$  (1 mark)
- iv. Show that
- $$Q = X_1(MPX_1) + X_2(MPX_2) + X_3(MPX_3) \quad (5 \text{ marks})$$
- c) The demand function for a commodity is
- $$P = 50 - 0.5q$$
- The cost of producing the commodity is made up of fixed cost of 200 shillings and variable cost of 0.2 shillings per unit. Find the profit function for the commodity (4 marks)

#### QUESTION FOUR (20 MARKS)

a) An open economy is defined by the following model

$$Y = C + I + G + X - M$$

$$C = c_0 + c_1 Y^d$$

$$T = t_0 + t_1 Y$$

$$M = m_0 + m_1 Y$$

$$I = I_0$$

$$G = G_0$$

$$X = X_0$$

- i. Find the equilibrium income (3 marks)
- ii. Find the equilibrium imports (2 marks)
- iii. What is the effect of a change in  $t_0$  on  $\bar{Y}$  and a change in  $m_0$  on  $\bar{Y}$  (2 marks)
- iv. Determine the export multiplier for the system (1 mark)

b) The utility of a consumer is defined by

$$U = Q_1^2 Q_2^2$$

Where  $U$  is the level of utility and  $Q_1$  and  $Q_2$  are the quantities of commodity 1 and commodity 2 consumed.

- i. The consumer initially purchases 5 units of  $Q_1$  and 2 units of  $Q_2$ . If he purchases 4 units of  $Q_2$ , how many units of  $Q_1$  will he purchase to maintain the same level of utility? (3 marks)
- ii. If he should decide to purchase 3 units of  $Q_1$ , how many units of  $Q_2$  will he purchase to maintain the same original level of utility? (3 marks)

c) The demand for commodity “a” is expressed as a function of some related commodity “b” as:

$$Q_a = 7 + (P_b)^{-\frac{1}{4}}$$

Find the cross-elasticity of demand for “a” with respect to price of “b” when  $P_b = 16$ .

Interpret your results (4 marks)

d) Explain the relationship between marginal propensity to consume and marginal propensity to save (2 marks)

**QUESTION FIVE (20 MARKS)**

- a) Among a group of students, 50 played cricket, 50 played hockey and 40 played volley ball. 15 played both cricket and hockey, 20 played both hockey and volley ball, 15 played cricket and volley ball and 10 played all three. If every student played at least one game, find the number of students and how many played only cricket, only hockey and only volley ball?  
(7 marks)
- b) You are given the following information about a firm
- $$P = 4 - \frac{1}{4}Q$$
- $$ATC = 0.05Q^2 - 0.3Q + 2 + \frac{4}{Q}$$
- i. Find the output level which will maximize the profits of the firm (5 marks)
- ii. Find ATC (2 marks)
- c) Clearly explain the need of Mathematics in Economics (6 marks)