MACHAKOS UNIVERSITY

University Examinations for 2020/2021 Academic Year

SCHOOL OF BUSINESS AND ECONOMICS

DEPARTMENT OF ECONOMICS

SECOND YEAR SPECIAL/SUPPLEMENTARY EXAMINATION FOR

BACHELOR OF ECONOMICS AND STATISTICS

BACHELOR OF ECONOMICS AND FINANCE

BACHELOR OF ECONOMICS

BACHELOR OF ARTS

EES 200: MATHEMATICS FOR ECONOMICS II

DATE: 26/3/2021 **TIME: 8.30-10.30 AM**

INSTRUCTIONS:

- **(i) Answer question one (Compulsory) and any other two questions**
- **(ii) Do not write on the question paper**
- **(iii) Show your workings clearly**

QUESTION ONE (30 MARKS)

a) Differentiate the following functions

i. $y = 5x^3 + \frac{2}{e^{x^2}}$ (2 marks)

ii. $y = \ln(5x^2 - 2)$ (2 marks)

b) The following input – output model shows the relationship between two industries; agriculture and manufacturing sectors in the economy for the year 2019.

	Agriculture	Manufacturing	Final demand	Total
Agriculture	100	50	360	510
Manufacturing	80	200	100	380

Required:

i. Calculate the input – output coefficients in this 2-sector industry (4 marks)

ii. If the final demand in the year 2013is given by: $\begin{bmatrix} 500 \\ 200 \end{bmatrix}$

Calculate the level of output required in each of the2 sectors of the economy (6 marks)

c) Find the product of the following two matrices (5 marks)

$$A = \begin{bmatrix} 8 & 1 & 3 \\ 4 & 0 & 1 \\ 6 & 0 & 3 \end{bmatrix} \qquad B = \begin{bmatrix} 10 & 3 \\ 7 & -1 \\ -6 & -7 \end{bmatrix}$$

d) Verify Euler's theorem for the function, $Q = L^3 + K^3 + L^2K$. (5 marks)

e) A monopolist producing two goods Q_1 and Q_2 has the following demand functions corresponding to each of the two goods: $P_1 = 56 - 4Q_1$ and $P_2 = 36 - 3Q_2$. Find the prices and output that maximizes total revenue for the firm (6 marks)

QUESTION TWO (20 MARKS)

a) Find $\frac{\partial y}{\partial_{x1}}$ and $\frac{\partial y}{\partial_{x2}}$ for the following functions and state the rule used

i) $y = (4x_1^3 + x_1x_2)(x_1^2 + 3)^3$ (3 marks)

ii) $y = \frac{6x_1 - x_2^3}{x_2^3}$ (3 marks)

b) The following information relates to national income model:

$$Y = C + I \quad C = 60 + 0.8Y \quad I = 50$$

Present this information in matrix algebra and then use matrix inversion method to solve for and $\overline{C}$ (5 marks)

c) A multiproduct firm is faced with the following cost function and a production constraint. The production constraint is constraint is stipulated in terms of a production quota.

Cost function: $C = 2Q_1^2 + 4Q_2^2 - Q_1Q_2 + 10$

Production quota: $Q_1 + Q_2 = 16$

i. Set up a constrained cost minimization problem and then construct the Langragian function (4 marks)

ii. Solve for the optimal values of output (5 marks)

QUESTION THREE (20 MARKS)

a) For some firm, the number of units produced when using x units of labour and y units of capital is given by the production function $f(x, y) = 80x^{0.25}y^{0.75}$

i. Find the equations for both marginal products (3 marks)

ii. What is the nature of the marginal products? (3 marks)

b) Given $U=(x+2)(y+1)$ and $P_x=4$, $P_y=6$ and $M=130$. Write the langragian function and then find the optimal levels x, y and λ (8 marks)

c) The demand function for a product is $P=81-x^2$ and the supply function is $P=x^2+4x+11$. Find the consumer's and producer's surplus (6 marks)

QUESTION FOUR (20 MARKS)

a) Given the function $Q=2k^2-3kL+L^2$, determine the nature of returns to scale for the function and interpret (4 marks)

b) Find the derivative of the following function: $y=f(x)=x^2e^x$ (4 marks)

c) For the following two demand functions, compute the partial elasticities of demand and state whether the commodities are substitutes or complements

$Q_1=7-2P_1-P_2$

$Q_2=23-P_1-3P_2$ (8 marks)

d) If $Z=f(x,y)=(x^2+4x-5y^3)^5$ find $\frac{\delta Z}{\delta x}$ and $\frac{\delta Z}{\delta y}$ (4 marks)

QUESTION FIVE (20 MARKS)

a) Prove Young's Theorem using the following function

$f(x_1,x_2,x_3)=2x_1^3+5x_1x_2^2-x_3^4$ (4 marks)

b) Evaluate the following $\int_3^6 \frac{x}{3x^2+4}dx$ (3 marks)

c) Find the minimum value of the function $Z=x^3+y^3+xy$ subject to the constraint $x+y-4=0$ (6 marks)

d) The demand and supply of two products are given by the functions

$Q_{d1}=10-2P_1+P_2$

$Q_{s1}=-2+3P_1$

$Q_{d2}=15+P_1-P_2$

$Q_{s2}=-1+2P_2$

Use cramer's rule to solve for the equilibrium prices and quantities (7 marks)