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# Application of extreme value theory in the estimation of value at risk in Kenyan stock market

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## ABSTRACT

Most financial institutions have faced a lot of losses due to the fluctuations of commodities prices. Traditionally normal distribution was applied and could not capture rare events which caused enormous losses. The objective is to estimate conditional quantiles of the returns of an asset which leads to Value at Risk directly using Extreme Value Theory which estimates the tails of the innovation distribution of financial returns. One of the most important approaches to risk management used in this study is quantification of risk using Value at Risk (VaR) which is achieved by Extreme Value Theory (EVT) that have the ability to estimate observations beyond the range of the data or out-of-sample data (extreme quantiles). Data from Nairobi Stock Exchange (NSE) specifically equities from Barclays Bank was applied at different confidence levels and it was observed that Peak-Over Threshold( POT) model of EVT and Generalized Pareto Distribution( GPD) which describes the tail of the financial returns captures the rare events which makes it the most robust method of estimating VaR.

Key words: Extreme Value Theory, Peak -Over Threshold (POT), Generalized Pareto Distribution (GPD).

## Introduction

Quantiles are used in moderate or high probability levels and also to high even to out-of-sample observations (beyond maximum observations). These are referred to as Extreme quantiles in Extreme Value Theory perspective. Extreme Value Theory explains significantly the limiting distribution of sample extremes in the same way normal distribution explains the limiting distribution of cumulative sums. This amounts to estimation of asymptotic distribution of extreme values without making any assumptions about an unknown parent distribution. An extreme movement relates directly to the tails of the distribution of the underlying data generating process. Extreme quantiles and the conditional ~ -quantiles of $Y_t$ are combined to obtain Value at Risk. The mathematical theory of EVT and its application in financial and insurance risk management were introduced by Embretchts (1997). Heavy-tailed, heteroskedastic and autoregressive models were used to estimate future market values of a portfolio. Conditional quantiles of innovations were estimated using Extreme Value Theory particularly the peak-over-threshold (POT) model. Estimation of the tails and quantiles (Extreme quantiles) of the returns were determined by fitting a Generalized Pareto Distribution (GPD) to the observations beyond a certain threshold.

Extreme Value Theory has the ability to quantify probabilistic behavior of large losses and able to adequately capture the tail behavior of stock returns. McNeil and Frey, (2000; Longin, (1997) and Mcneil, (1998) used estimation based on limit theorems for block maxima to estimate extreme returns. Danielsson and de Vries, (1997a, b) used semi-parametric approach based on the Hill-estimator to estimate extreme returns for the tail index and bases on conditional normality which is not well suited for estimating the distribution of large quantiles of the Profit and Loss distribution. Barone-Adesi, Bourgoin and Giannopoulos (1998) fitted a Garch-model to a fanancial returns series and uses historical simulation to infer the distribution of the residuals. Their results work well for a large data set and as the data becomes smaller it might not work well in the

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estimation of the tails of the residuals and this requires other methods like EVT. Embretchts Resnick and Samorodnitsky, (1999) advocated the fully parametric estimation technique which is based on a limit result for the excess-distribution over high thresholds or Peak-Over-Threshold (POT) i.e. the Generalized Pareto Distribution (GPD). Mcneil and Frey, (2000) estimated conditional quantiles by combining GARCH Models for forecasting volatility and EVT techniques applied to the residuals from the GARCH analysis. The disadvantage for this method is that the results of the EVT analysis will be sensitive to the fitting of a GARCH model to the entire data set in the first stage. Gencay and Selcuk, (2004) reviewed estimation of VaR in some emerging markets using various models including EVT. From their emperical results it showed that EVT- based models provides more accurate VaR especially in a higher quantiles. Harmantzis et. al. (2005) and Marinelli et. al., (2006) presented the performance of EVT in VaR and ES estimation compared to the Gaussian and Historical simulation models together with other heavy-tailed approach. From their results it was found that fat – tailed models can predict risk more accurately than non-fat-tailed ones and there exists the benefits of EVT framework especially method using GPD.

The key approach to EVT is that the distribution of extreme returns converges asymptotically to a particular known distribution rather than a single parametric method where the data is fitted to a whole distribution. Because of stochastic volatility in financial data, fitting the whole data to a single parametric model may not be appropriate when applied directly (Mwita, (2003)). Danielsson (1998) showed that EVT does not capture the risk well when the probabilities are as low as 0.95. Another very significant approach is to combine EVT which is used to estimate the residuals and Artificial Neural Network which is to estimate the conditional volatility and conditional expectations of returns Diagne, (2003).In this approach EVT method is used to estimate the quantiles of the returns directly.

# Methodology

Extreme Value theory is the most robust method in the estimation of the tail behaviour of a distribution. EVT models the tail returns and EVT estimates measures the risks of financial returns.

## Distribution of Exceedances

The framework of estimating the distribution of excesses over certain threshold point, which identifies the starting of the tail, is based on Peak-Over- Threshold (POT) approach. This is generally considered to be most applicable in practical applications since it is efficient in the use of limited data on extreme values.

## The Generalized Pareto distribution (GPD)

The distribution function $G_{\psi,\beta}$ is defined as follows;

$$G_{\cdot,\text{¶}}(\pounds)=\begin{cases}1-\left[1+\frac{\cdot\,\pounds}{\text{¶}}\right]^{-\frac{1}{\cdot}} & \text{if } \cdot\neq 0\\ 1-\exp\left[-\frac{\pounds}{\text{¶}}\right] & \text{if } \cdot=0\end{cases}\qquad(1)$$

The definition of Generalized Pareto distribution is given under the following conditions:

Given that ¶ > 0, and the support $\pounds \geq 0$ when $\psi \geq 0$ and $0 \leq \pounds \leq -\text{¶}/\cdot$ when $\cdot < 0$.

$F_{\varepsilon}$ is a class of distribution which falls into three categories depending on the value of the shape parameter · in the limiting GPD approximation to the distribution of excesses. If · > 0 then the distribution corresponds to heavy-tailed distributions where the tails decays like a power functions, these distributions includes Pareto, Student t, Cauchy, Burr, Loggamma, and Frechet distributions. If · = 0 then the distributions includes the normal, exponential, gamma, and lognormal, whose tails decay essentially exponentially. When · < 0, then the distributions are short tailed distributions with finite right endpoint, such as the uniform and Beta distributions. Since these distributions subsume other distributions under common parametric form, they are referred to as generalized distributions. When · > 0, reparametrized type of the usual Pareto distribution with shape $\alpha = 1/\cdot$ is obtained. If · < 0, then type Π Pareto distribution is obtained, if · = 0, it gives the exponential distribution. If the family of distribution is extended by adding a location parameter μ, then GPD ($G_{\cdot,u,\text{¶}}(\pounds)$) is defined to be $G_{\cdot,\text{¶}}(\pounds-u)$.

## Theorem: Pickands-Balkema-Gnedenko-de Haan

Let $x_1, x_2, x_3 \ldots\ldots x_n$ be n independent realizations of a random variable X having a distribution function F(x). For a large class of underlying distribution functions F the conditional excess distribution function $F_u(y)$ given an appropriately high threshold u is approximated by

$$\lim_{£\to\infty}\left\{ \sup_{0\le £\le £_0-u}\left|F_u(x)-G_{\cdot,¶(u)}\right| \right\}=0 \quad (2)$$

Where $\varepsilon_0$ is defined by $\varepsilon_0$ = sup $\{\varepsilon\in\Re$ such that F (x) <1}

From this theorem it suggests that as the threshold u is progressively increased, the excess distribution ($F_u$) over the threshold may be approximated by Generalized Pareto distribution (GPD) for some · and ¶ provided the underlying distribution F satisfies the extremal-types theorem. Excesses over the threshold $N_u$ are assumed to be iid with exact GPD parameters.

## Estimating Excess Distribution (Unconditional Quantiles)

Let $£_t$ be an iid random variables and also let $y_1$, $y_2$,….$y_N$ , ~ gives a series of exceedances over the threshold $u = q^{£}_{\sim}$ . The definition of the distribution of excess losses over a high threshold u is given as;

$$F_u(y) = \Pr(£_t) \le u + y \mid £_t > u \quad , \quad y > 0 \quad (3)$$

The assumption is that the excesses are iid having distribution function $F_u$ and u is less than the right-hand end point of the distribution $e_F = \sup_{£\in\Re}\{F(£)\} < 1$. In terms of underlying loss distribution 3, the distribution of excess losses over a high threshold may be defined as;

$$F_u(y) = \frac{F(£ + y) - F(u)}{1 - F(u)} \quad (4)$$

When equation 4 is rearranged, the tail distribution of the random variable $£_t$ above threshold u is obtained as

$$\overline{F}(u + y) = \overline{F}(u).\overline{F}_u(y) \quad (5)$$

The tail of original distribution may be easily estimated by estimating F and $F_u$ separately. The Peak-Over Threshold may now be used to model observations exceeding a high threshold u. From Pickands-Balkema-de Haan theorem, it is stated that $F_u(y) \to G_{\cdot,¶(u)}(y)$

(6)

Combining equations (5) and (6) and setting $\varepsilon$ = u + y, the model can also be re-written as

$$F(x) = (1 - F(u))G_{\cdot,¶}(x - u) + F(u) \quad \text{for } x > u \quad (7)$$

This formula shows that the model may be interpreted in terms of the tail of the underlying distribution F(x) for x > u. Equation7 is used to construct a tail estimator after estimating F(u).

Method of Historical Simulation (HS) taking the empirical estimate F (u) as n-Nu/n

The maximum likelihood estimators $\hat{\cdot}_n$ *and* $\hat{¶}_n(u)$ of the GPD parameters are fitted with the residual excess sample $\hat{£}_t$ defined as $\hat{£}_t = \hat{Y}_t - \hat{q}^t_{\sim}(X_t)$, assuming that $\hat{£}_t$ is iid then the tail estimator formula is given by

$$\hat{F}(x) \cong F_n u(x) = \left[1 - \frac{N_u}{n}\right] \times \left[1 + \frac{\hat{\cdot}_n(x - u)}{\hat{¶}_n(u)}\right]^{-1/\hat{\cdot}} \quad (8)$$

## Estimation of Value at Risk

The quantile estimate which is an unknown parameter of an unknown underlying distribution may be calculated by inverting the tail estimation equation 8 to obtain the Value at Risk which is given by

$$\hat{q}_{\sim}(u) \cong \frac{\hat{¶}_n(u)}{\hat{\cdot}_n}\left\{\left[\frac{n}{N_u}(1 - \sim)\right]^{\hat{\cdot}_n} - 1\right\} + u \quad (9)$$

This is the unconditional quantile estimate of an underlying distribution.

The GPD is fitted to the $N_u$ excesses to obtain the estimates by assuming that number $N_u$ out of a total n data points exceed the threshold. The Maximum likelihood estimation (MLE) method is used to obtain parameters, where parameter values are chosen to maximize the joint probability density of the observations. This method help to give estimates of statistical error (standard errors) for parameter estimates which makes it a general fitting method in statistics. From Hosking, J., & Wallis, J. (1987), (MLE) is not as efficient as the method of moments even in samples as large as 500. GPD estimators based on the method of moments are of the form.

$$\hat{\cdot} = \frac{1}{2}\left(1 - \frac{\overline{z} - u}{s^2}\right) \quad \textit{and} \quad \hat{¶}(u) = \frac{\overline{z} - u}{2}\left(\frac{\overline{z} - u}{s^2} + 1\right) \quad (10)$$

Where $\overline{z}$ and $s^2$ are empirical mean and variance respectively.

Equation 10 assumes that the unknown distribution function F has an exact GPD tail.

A more realistic approach to use for any heavy tailed distribution F is to relax the exact type of distribution by using result of Balkema-de Haan and Pickands.

It can be seen from (16) that the estimates $\hat{\cdot}_n(u)$ *and* $\hat{\cdot}_n$ are consistent and asymptotically normal an $N \to \infty$ and for $\hat{\cdot}_n > -\frac{1}{2}$. Smith, R. (1987), also obtains asymptotic normality results of the estimates under the weaker assumption that the excesses are iid from $F_u(y)$ which is approximately GPD.

The log-likelihood function for the estimates is given as

$$L(\cdot_n, ¶_n(u)) = N_u \log(¶_n(u)) - \left(\frac{1}{\cdot_n} + 1\right)\sum_{i=1}^{N_u} \log\left(1 + \frac{\cdot_n y_i}{¶_n(u)}\right)$$

## Estimating Tails of Distributions of Conditional Quantiles for Dependent Data

In order to obtain appropriate results from extreme value theory, independence in the series is required. Let us denote the filtered excess residuals by

$$£_t^+ = \hat{Y}_t - \hat{q}_{\sim}^t(X_t) > 0 \qquad t = 1,2,\ldots,n \qquad (11)$$

Assumption of independence are relaxed up to some high level of $\sim$. The approach here assumes independence for only $£_t^+$ corresponding to large $\sim$.

If $f_{X_t}(£)$ is a conditional density function of $\hat{£}_t$ on $X_t$, then the following assumptions hold for filtered excess residuals.

## Assumption 1

Assume that $\hat{£}_t = Y_t - \hat{q}_t^{\sim}(X_t)$ are sample residuals approximating $£_t$. At least to a good approximation, the conditional density function $g_{xi}(£)$ of the excesses of $\hat{£}_t$ over the threshold $q_{\sim}^{£}$ given $X_t = x_t$ is assumed such that

$$g_{x_i}(£) = g(£) \; \textit{for all } x_i \; \textit{and} \; G(£) \in MDA(H_{\cdot}, \cdot > 0) \qquad (12)$$

From the definition of $£_t$ as iid, the condition states that the excess conditional distribution of filtered excess residuals are heavy tailed and independent of the covariate beyond the threshold at high probability.

The main objective is to find the distribution function of the data above the threshold $u = q_{\sim}^{£} = 0$ whose inverse will be Quantile Autoregression -scaled extreme quantile. From 4.3 the implicit form of the distribution may be written as

$$F(q_{\sim}^{£} + £) = F(q_{\sim}^{£}) + (1 - F(q_{\sim}^{£}))F_{q_{\sim}^{£}}(£) \qquad (13)$$

Where its estimate is given as

$$\hat{F}(\hat{q}_{\sim}^{£} + £) = F(\hat{q}_{\sim}^{£}) + (1 - \hat{F}(\hat{q}_{\sim}^{£}))\hat{F}_{\hat{q}_{\sim}^{£}}(£)$$

$\hat{F}(\hat{q}_{\sim}^{£}) = F(q_{\sim}^{£}) = \sim$ from Cai, Z and Roussas (1997) which implies that

$\hat{F}(\hat{q}_{\sim}^{£} + £) = \sim + (1 - \sim)\hat{F}_{\hat{q}_{\sim}^{£}}(£)$ which simplifies the tail estimator to

$$\hat{F}(\hat{q}_{\sim}^{£} + £) = \sim + (1 - \sim)\hat{F}_{\hat{q}_{\sim}^{£}}(£)$$
$$\Leftrightarrow \hat{F}(£) = \sim + (1 - \sim)\hat{F}_0(£) \qquad (14)$$

Where $£ > 0$ *and* $q_{\sim}^{£} = 0$

A lemma which is proved in (Mwita, 2003) shows that $\hat{F}(£)$ is asymptotically a generalized Pareto distribution function estimator.

Part of the data based on relatively high probability level say $\alpha$ is assumed to be an initial as well as the beginning of the right-hand tail of a heavy tailed distribution. A high probability levels, say $\varphi > \alpha$ is taken to obtain extreme quantiles. First consider the iid random variables $\varepsilon_t$ … based on the process given in equation 1. A high quantile $q_\alpha$ is considered in order to derive an estimate of an extreme quantile $q_\varphi$ for $\varphi \approx 1$,

where $\alpha < \varphi$ is large but not as close to 1 as $\varphi$.

It should be observed that the assumptions of independence are related up to some high levels of $\alpha$. A method which involves historical simulation (HS) in finding the threshold assumes standardized excesses over the conditional mean are iid, this kind of approach assumes independence for only $\varepsilon_t^+$ corresponding to large $\alpha$. Assuming $q_{\alpha,\varphi}$ to be the quantiles above a threshold $q_{\sim}^{£} = 0$ based on $\varepsilon_t$ and derived by inverting the distribution $F(\varepsilon)$ at a particular level of $\varphi > \alpha$. That is for fixed $\varphi \in (0,1)$,

$$q_{\cdot\cdot}^{£} = \inf_{£>0}\{F(£) \geq \cdot\cdot\} \quad = \inf_{£>0}\{1-(1-\sim)(1-F_0(£)) \geq \cdot\cdot\}$$

*from* 3.24

$$\approx \operatorname{Sup}_{£>0}\left\{\overline{G}_{\cdot\,(\sim)}(£) \leq \frac{1-\cdot\cdot}{1-\sim}\right\}, \;\sim \rightarrow$$

$$1 \text{ and } \overline{G} = 1 - G \qquad = \overline{G}_{\cdot\,(\sim)}^{-1}\left(\frac{1-\cdot\cdot}{1-\sim}\right)$$

$$= \frac{(\sim)}{\cdot}\left(\left(\frac{1-\cdot\cdot}{1-\sim}\right)^{-\cdot} - 1\right)$$

(15)

Its estimate is denoted by $q_{\cdot\cdot}^{£} = \inf_{£>0}\left\{\hat{F}(q_{\sim}^{£} + £) \geq \cdot\cdot\right\} = \frac{\hat{}(\sim)}{\cdot\hat{}}\left(\left(\frac{1-\cdot\cdot}{1-\sim}\right)^{-\cdot\hat{}} - 1\right)$.

$\hat{q}_{\cdot\cdot}^{£}$ is a consistent estimate of $q_{\cdot\cdot}^{£}$ since $\sqrt{N_{\sim}}\left(\cdot\hat{} - \cdot\,,\; \frac{\hat{¶}(\sim)}{¶(\sim)} - 1\right)$ is consistent and

asymptotically normal with mean zero covariance given in lemma 4.1 in Mwita, (2003), clearly

$$\left(\cdot\hat{} - \cdot\,,\; \hat{¶}(\sim) - ¶(\sim)\right) \xrightarrow{p} (0,0) \; as \sim \rightarrow 1 \; and \; N_{\sim} \rightarrow \infty$$

.

Therefore $\frac{\hat{¶}(\sim)}{\cdot\hat{}}\left(\left(\frac{1-\cdot\cdot}{1-\sim}\right)^{-\cdot\hat{}} - 1\right)$ estimates

$\overline{G}_{\cdot\,,¶(\sim)}^{-1}\left(\frac{1-\cdot\cdot}{1-\sim}\right)$ consistently which is proved in Dudewicz and Mishra, (1998).

In choosing on appropriate threshold, we are faced with a bias-variance tradeoff. Theoretically the threshold should be as high as possible for the Pickands-Balkema-de Haan theorem to be satisfied. In practice too high threshold means that we are remaining with very few observations above the threshold for estimating GPD parameters leading to very high variance of estimates (McNeil and Frey (2000)). Note that if by visual judgment it is observed that the required quantiles does not fall in the peak area, we use other appropriate method. That is, a series of iid random variables may be generated from normal or t distribution if it is observed that the required level of $\alpha$(in this case$\varphi$) does fall in appropriate region. A set of quantiles is then fitted on the excesses over the quantiles adjusted random variables (Mwita, (2003).

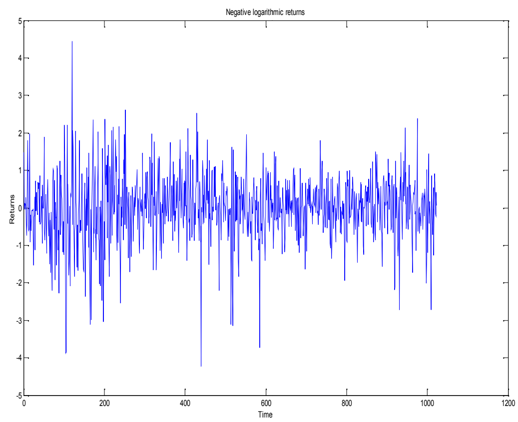
# Results and Discussion

## Estimation of Extreme Quantiles and Value at Risk

In this section data analysis for Extreme value applications in MATLAB is demonstrated. This entails the estimation of the distribution of excesses over a certain high threshold using the POT approach which actually identifies the starting point of the tail. The data used is stock market data specifically equities from Barclays Bank of Kenya consisting of 1023 observations. (15)

Logarithmic transformation was performed on the raw data to obtain logarithmic returns which then applied in data analysis. Figure 1 below shows logarithmic returns plotted against time.

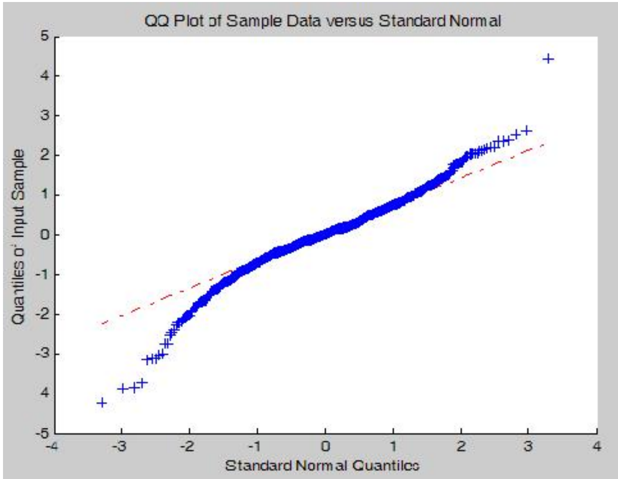
Figure - 1 Daily negative logarithmic returns




After logarithmic transformation, the mean of the data became stationary. From figure 1 it is observed that there is presence of values clustering. This actually means that small values follows small values and similarly large values follows large values which indicates presence of short-range dependence, resulting into doubting the assumption independent and identically distributed returns that may be violated due to the observed clustering. The presence of clustering of returns indicates the presence of stochastic volatility. Modeling the tails of a distribution with GPD requires observations to be approximately independent and identically distributed (iid). To produce a series of iid observations, GARCH-type stochastic volatility model is fitted to the returns and the filtered residuals and Volatilities are extracted from the returns of the equity prices. The residuals obtained after model filtration from each return series are standardized to have a Zero – mean, unit-variance and also iid of which are now used in EVT estimation of the sample CDF tails. Value at Risk can now be obtained using equation (3.15).

The first step in the use of Extreme Value Theory analysis is to examine the QQ-plots and the distribution of mean of excesses. Most financial data series are fat tailed and therefore the graph makes it possible to assess their goodness of fit of the data series to the parametric model. The fat-tailness of a distribution is confirmed by the use of QQ-plot, which should be concave in nature to indicate a fat-tailed distribution. This can be shown in figure 2.

Figure -2 QQ Plots




If the graph follows a linear form, then the parametric model fits the data well. It is possible for the QQ-plots to assist to detect outliers if the distribution of the data is known. QQ-plots also makes possible to assess how well the given model fits the tail of the empirical distribution. From figure 2, it is shown that the graph is curved to the top at the right end or to the bottom at the left end indicating that the empirical data is fat tailed.

Figure- 3 Mean Excess Plot

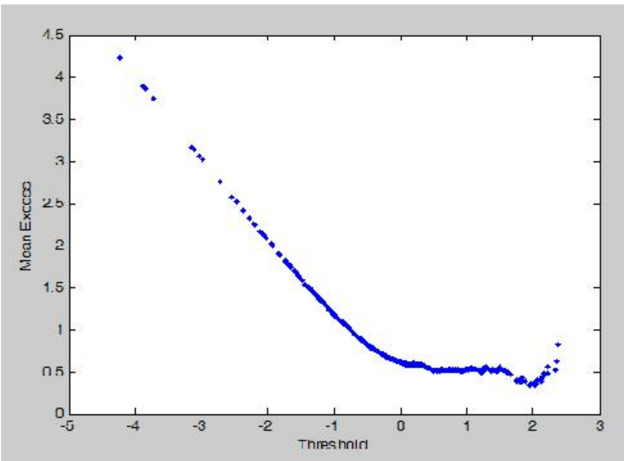



Figure 3 shows a plot of sample mean excesses of a stock data against different thresholds. This is a plot of some few sample mean excesses. This is because plotting all the excesses affect and finally distorts the plotting. From figure 9, it is observed that the graph declines and begins an upward trend which indicates the presence of a heavy-tailed distribution. A threshold is chosen observing the area with a linear shape on the graph straightening upwards. It is observed that the graph begins to straighten upward around threshold 2 and also the function tends to infinity like a GPD, which provides a reasonable fit to the whole data set. The chosen threshold is 2.00 meaning that from the actual data 50 out of 1023 data points exceed the threshold.

## Estimations at the Tail of the Distribution

The 50 exceedances over high threshold are fitted to the GPD using Maximum Likelihood Estimation (MLE). The quantile (percentile) value at the tail from estimated parameters of different distributions is estimated. Basing on these data, the parameter estimates are $\hat{\xi} = 0.3582$ and $\hat{\beta} = 0.2212$. The shape parameter $\xi > 0$ is an indication of a heavy-tailed distribution. This can be interpreted to mean, the higher the value of the shape parameter the heavier is the tail and the higher the derived quantile estimates and the corresponding prices. At this point quantile estimates and confidence intervals for a high quantiles above the threshold in a GPD are calculated.

Figure - 4 Exceedance distributions

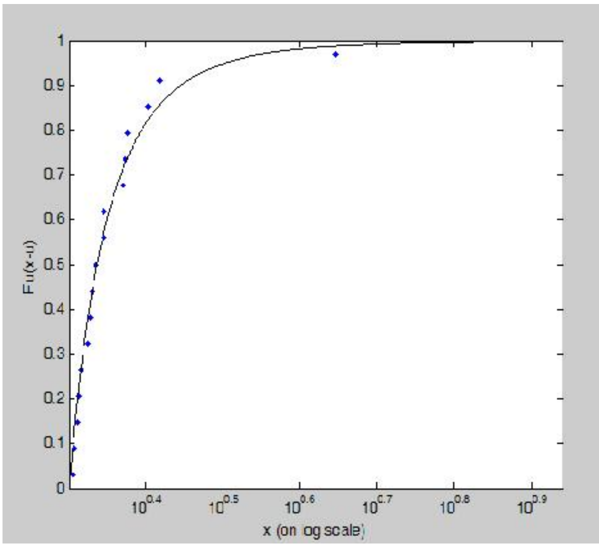



Figure 4 shows a smooth curve which is the estimated GPD Model for the excess distribution and the points which shows the emperical distribution of 50 extreme values showing that GPD model fits the excess losses well.

From Figure 5, y-axis indictes the tail probabilitues 1 – F(x). The Threshold of 2.00 corresponds to a tail probability of approximately 0.05 which can be observed from the top left corner of the graph. The solid curve indicates the tail estimation which can be extrapolated to areas of sparse data and the points again shows the 50 large losses.

Figure - 5 Tail estimate

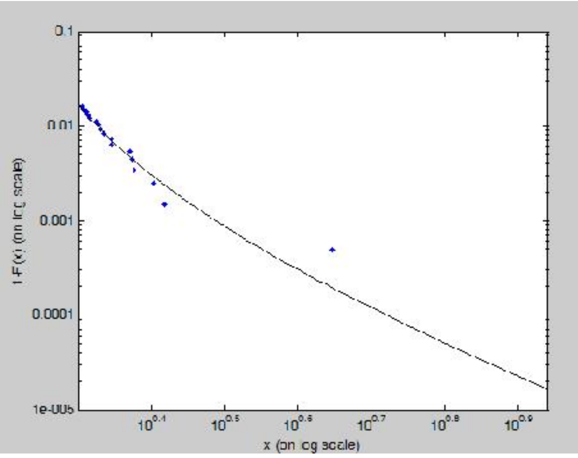
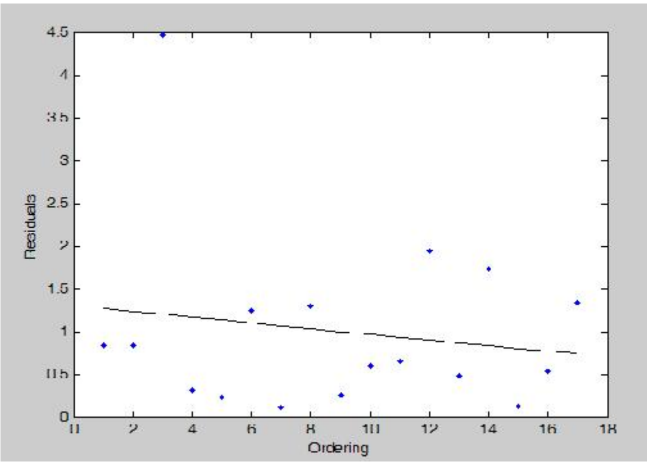



Figure 6 shows the scatter plot of residuals from a Generalized Pareto Distribution fitted to stock market data over a high threshold 2.The solid line observed is the smooth of the scattered residuals.

Figure - 6 Scatter plot of residuals




In Figure 7, Value at Risk is esimated to be $VaR_{0.999} = 3.0726$ which is calculated using equation 3.20 and is statistically refered to as a quantile estimate. The figure shows the point and interval estimation of VaR at the 0.999[th] quantile and 95 percent confidence intervals for the stock market data. Confidence interval of VaR yields an asymptotic interval where VaR lies and these demonstrate a fundamental asymmetry in the estimation of a high quantile for heavy-tailed data. Where the vertical red dotted line intersects with the tail estimate gives the point Value at Risk of 3.0726. The left vertical dotted blue line shows the lower confidence level which is 2.5822 and the right vertical dotted blue line shows the upper confidence level which is 6.6548 for the Value at Risk and the Horizontal red thick line corresponds to the 99.9% confidence level. The advantage of estimating VaR using GPD method is that, this method can estimate VaR outside the sampling interval

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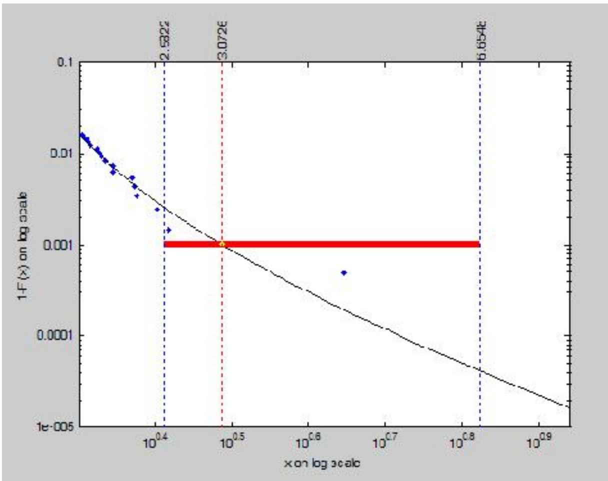
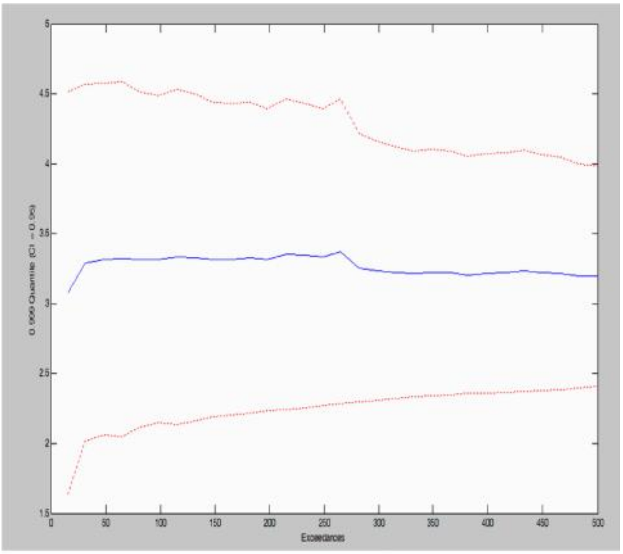
Figure-7 Point estimate at the tail




The left vertical dotted blue line shows the lower confidence level which is 2.5822 and the right vertical dotted blue line shows the upper confidence level which is 6.6548 for the Value at Risk and the Horizontal red thick line corresponds to the 99.9% confidence level. The advantage of estimating VaR using GPD method is that, this method can estimate VaR outside the sampling interval.

In figure 8, the tail estimates with 95 percent confidence intervals are plotted against the number of exceedances. The dotted lines are the upper and lower 95 percent confidence intervals. This actually describes variability of the estimate of a high quantile in the tail of the data based on GPD estimation with varying threshold or number of exceedances.

Figure - 8 Stock market data loss estimates of 0.999-quantile as a function of exceedances




## Estimating Extreme Quantiles Considering Volatility of Returns

This approach entails the estimation of Value at Risk, where the main aim is to find out the possible extent of a loss arising from adverse market movement over the next day taking into account current volatility background. Volatilities are extracted using the GARCH model as explained in the previous section. In a period of high volatility an extreme value appears less extreme than the same value in a period of low volatility. This section shows how this dynamic risk measure of market risk procedure can be improved by using EVT to take into account the extreme risk over and above volatility risk.

Figure 9 shows the extracted volatilities derived from the output function of GARCH Model. After the identification of the shape parameters, it is now possible to estimate extreme quantiles which indicates the scale losses if the threshold were to be exceeded.

Figure - 9 Volatility

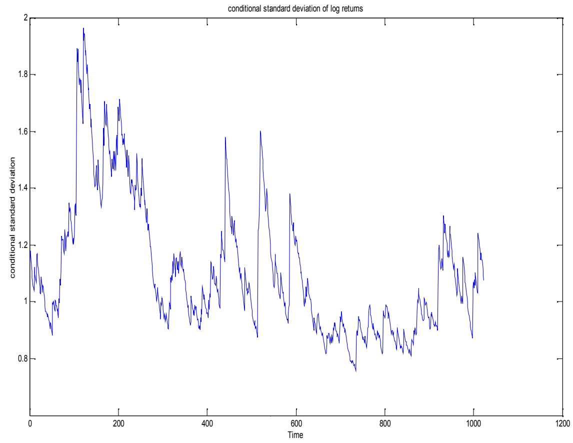
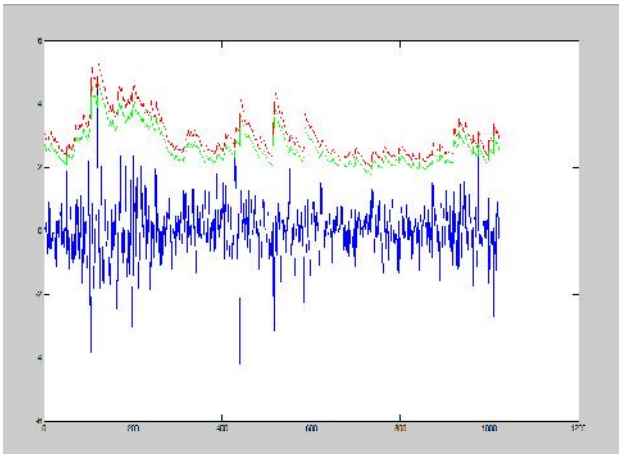



Figure - 10 Value at Risk at different confidence intervals



From figure 10 the green line gives quantiles at confidence interval of 0.95, the Red line shows extreme quantiles at confidence interval of 0.99 superimposed on the logarithmic returns of stock market data. This is called conditional quantiles or The Value at Risk which should be observed from time to time to avoid excessive losses.

It can be observed in figure 10 that at high probability level, Value at Risk (conditional quantiles) can be captured properly using Extreme value Theory specifically the POT method.

## Computing VaR and ES using Extreme Value Theory Method

Alternatively equation (9) is used to calculate Value at Risk. Using the threshold of 2, number of observations above the threshold is 50, total number of observations as 1023, Shape parameter as 0.3582, Scaling parameter as 0.2212 and varying confidence level, the results may be obtained as follows.

Table - 1 Point Estimates for VaR and ES

| Level of confidence | 0.95 | 0.99 | 0.995 | 0.999 |
|---|---|---|---|---|
| · | 0.3582 | 0.3582 | 0.3582 | 0.3582 |
| $\hat{¶}$ | 0.2212 | 0.2212 | 0.2212 | 0.2212 |
| *VaRq* | 2.5105 | 2.5330 | 2.5358 | 2.5380 |

Looking at the VaR from table 1, with 5% confidence level tomorrows loss (left-tail) for the stock market prices to exceed 2.5105%. Analogously the same interpretation holds for 1%, 0.5% and 0.1%. In practice when the portfolio loss is known, then precautions can be taken to mitigate against it.

## Conclusion

VaR estimates were obtained using Extreme Value method. In the estimation of extreme quntiles, the distribution of excesses over a certain high threshold was based on Peak-Over-Threshold method which identified the starting of the tail. After the excesses over a high threshold were fitted to the GPD, parameters were estimated which were used to estimate Value at Risk. The point estimates and interval estimates of VaR at 99.9% and 95% confidence intervals for the equities were clearly shown and found to capture the financial Risks significantly since this method can estimate VaR outside the sampling interval.

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