

DATE: 20/1/2021

TIME: 8.30-10.30 AM

INSTRUCTIONS:

- (i) Answer question one (Compulsory) and any other two questions
- (ii) Do not write on the question paper
- (iii) Show your workings clearly

QUESTION ONE (COMPULSORY) (30 MARKS)

- a) Clearly distinguish between the following pairs of concepts. Make use of illustrations and diagrams as much as you can:
 - i. Production Possibilities Set and Input requirement Set (3 marks)
 - ii. Roy's Identity and Shepherds Lemma (3 marks)
 - iii. Compensated Demand functions and Uncompensated Demand functions (3 marks)
 - iv. Cournot's equilibrium and Stackelberg's equilibrium (3 marks)
- b) In a duopoly market, the market demand and cost functions of the firms are given as:

P = 100-0.5X , where $X = X_1\text{+}X_2$

 $C_1 = 5X_1$

 $C_2 = 0.5 X_2^2$

i. If firm 1 is a quantity leader, determine the equilibrium price and quantities in the market (4 marks)

ii. Suppose firm 1 is a price leader, how would the answers in (i) above change?

(4 marks)

- iii. Suppose the two firms decided to form a cartel and maximize their joint profits, determine the equilibrium price, quantities and profit in the market (6 marks)
- c) Explain **four** properties of a legitimate production function (4 marks)

QUESTION TWO (20 MARKS)

A consumer's indirect utility function is given as:

$$V(P,M) = \left(P_1^2 + P_2^2\right)^{0.5} M$$

Where P_1 and P_2 are prices of goods X_1 and X_2 , respectively and M is the consumer's Income.

a) Derive the following:

- ii. The uncompensated demand function for good X_1 (4 marks)
- iii. The uncompensated demand function for good X_2 (3 marks)

b) Using the functions above, state and demonstrate the slutsky's equation. (10 marks)

QUESTION THREE (20 MARKS)

a) Consider an industry with the following structure. There are 50 firms that behave in a competitive manner and have identical cost functions given by:

$$c(y) = \frac{y^2}{2}$$

The demand for the product is given by D(P) = 1000 - 50P

- i. What is the total supply from the market? (3 marks)
- ii. What is the equilibrium price and quantity in the competitive market? (2 marks)
- b) What is elasticity of substitution? Compute the elasticity of substitution for the following function. (5 marks)

$$Q = AL^{\alpha}K^{\beta}$$

c) Given the firms production function as $y = AL^{\alpha}K^{1-\alpha}$

Let *w* be the price of labour and *r* the price of capital. So that the firm's expression of the cost equation is given as C = wL + rK

Derive the corresponding cost function.

(10 marks)

QUESTION FOUR (20 MARKS)

- a) A short run production function is given as $Q = X^{0.5}$, where Q is the output and X is the input. Is the production function concave? Show your working. (4 marks)
- b) Let *p* represent the output price and *w* represent the input price. Derive the firm's profit function. (8 marks)
- c) Is the profit function derived legitimate? Show your working. (8 marks)

QUESTION FIVE (20 MARKS)

a) Given the following cost function,:

 $C(w_1, w_2, y) = 10w_1^{\frac{1}{3}}w_2^{\frac{2}{3}}y$, where y is the output and w_1 and w_2 are the prices of two inputs x_1 and x_2 respectively. Check whether the cost function satisfies the properties of a cost function. (7 marks)

b) From the cost function in a) determine the underlying production function. (7 marks)

c) Given the indirect utility function
$$V(p,m) = \frac{m^2}{4p_1p_2}$$

Demonstrate the properties of indirect utility functions. (6 marks)