MACHAKOS UNIVERSITY

University Examinations for 2021/2022 Academic Year

SCHOOL OF BUSINESS AND ECONOMICS

DEPARTMENT OF ECONOMICS

SECOND YEAR ………… SEMESTER EXAMINATION FOR

ECONOMICS AND FINANCE

ECONOMICS AND STATISTICS

BACHELOR OF ARTS

EES 200: MATHEMATICS FOR ECONOMICS II

DATE: **TIME:**

INSTRUCTIONS:

(i) Answer question one (COMPULSORY) and any other two questions
(ii) Do not write on the question paper
(iii) Show your workings clearly

QUESTION ONE (COMPULSORY) (30 MARKS)

a) Determine the nature of returns to scale for the following functions: (4 marks)
   i) $Q = 10K^{2/5}L^{3/5}$
   ii) $Q = 15K^{0.5}L^{0.4}$

b) Compute the consumer and producer surplus given the following information: (6 marks)

   $P = 50 - 0.5Q;$ $P_e = 30$

   $P = 13 - Q^2;$ $P_e = 4$

c) A firm manufactures three types of pans; round bottom, square bottom and triangular bottom. These products use three available inputs from material; man hour and machine hour as shown in the table below:

Inputs	Per unit utilization of inputs			Availability
	Round bottom	Square bottom	Triangular bottom	
Raw material	6	5	3	110
Man-hour	8	4	5	130
Machine hour	7	6	3	125

Using matrix algebra, determine the number of units of each product to produce in order to utilize completely the available resources. (6 marks)

d) A consumer's utility function is given as:
$U = 3xy$
Suppose the price of good x is \$4 and that of good y is \$2 and the consumer has \$24 dollars to spend on the two goods:
Required:
i) Obtain the values of x and y that will maximize the consumer's utility. (4 marks)
ii) Determine whether the values obtained above yield maximum utility or not. (4 marks)

e) Compute the derivatives of the following functions, clearly stating the rules used:
i) $y = 5e^{2x^3-7x^2}$ (3 marks)
ii) $y = 8^{3x+5}e^{5x+9}$ (3 marks)

QUESTION TWO (20 MARKS)

a) Suppose that the population of Nairobi County is growing according to the function:

$N(t) = 2{,}400{,}000e^{0.03t}$, where N(t) is population size at time t; t is the number of years.

What time is required for the population to double? (4 marks)

b) Evaluate: (4 marks)

$$\int_1^2 \frac{2-6x}{(2x-3x^2)^3}\,dx$$

c) Given the following information of a 3-sector economy:

		Input to				
		Agriculture	**Manufacturing**	**Services**	**Final demand**	**Total Output**
Output From	**Agriculture**	300	450	250	200	1200
	Manufacturing	420	500	280	600	1800
	Service	340	0	60	200	600
Other inputs		140	850	10		
Total inputs		1200	1800	600		

Required:

i) The input-output matrix (3 marks)

ii) The Leontief Matrix (2 marks)

iii) The output requirement for each sector given that the final demand changes to $D = \begin{pmatrix} 600 \\ 1800 \\ 600 \end{pmatrix}$ (7 marks)

QUESTION THREE (20 MARKS)

a) Find f_{xy} and f_{yx} for each of the following functions (4 marks)

(i) $f(x, y) = 3x^2 y + 6x^2 + 2y^3$

(ii) $f(x, y) = e^{2y} y^3 + \ln(x^2 y^2)$

b) Find the elasticity of demand for the following hyperbolic demand function: (3 marks)

$$Q = 10P^{-2}$$

c) You are given the following national income model:

$$Y = C + I + G$$
$$C = 120 + 0.8Y$$
$$I = 100 + 0.1Y$$
$$G = 300$$

Required:

i) Present the model in matrix format (2 marks)

ii) Using Cramer's rule, find the equilibrium values of Y, C and I. (6 marks)

d) Assuming that the rate of net investment flow is given by the following function and that $K(0) = 30$: $I(t) = 24t^{\frac{2}{5}}$

i) Find the time path of capital stock $K(t)$ (2 marks)

ii) Find the amount of capital formation over the interval [1,4] (3 marks)

QUESTION FOUR (20 MARKS)

a) Given the following national income model:

$$Y = C + I_0 + G_0$$

$$C = c_0 + c_1(Y^d) \text{ where } c_0 > 0 \quad 0 < c_1 < 1$$

$$T = t_0 + t_1 Y \quad t_0 > 0 \quad 0 < t_1 < 1$$

$$Y^d = Y - T$$

Compute and interpret the tax rate multiplier (4 marks)

b) A firm produces jam from two raw materials, strawberries and black berries. The production function for the jam is given as: $Q = 20x^{0.2}y^{0.8}$ where x is strawberries and y is blackberries. If each unit of straw berries cost 20 shillings and each unit of blackberries cost 2 shillings, and the firm has only 1450 shillings at its disposal:

i) Determine the optimal values of strawberries and blackberries to be bought and the maximum amount of jam to be produced. (7 marks)

ii) Show that $\frac{MPx}{MPy} = \frac{Px}{Py}$ at the maximum output (5 marks)

c) Compute the product AB and BA for the following matrices (4 marks)

$$A = \begin{bmatrix} 4 & 9 & 10 \\ 5 & 2 & 0 \\ 6 & 1 & 6 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 15 & 4 \\ 3 & 2 & 7 \\ 8 & 3 & 5 \end{bmatrix}$$

QUESTION FIVE (20 MARKS)

a) Prove Euler's Theorem given the following production functions:

i) $Q = 40K^{0.5}L^{0.5}$ (3 marks)

ii) $Q = 100K^{0.4}L^{0.5}$ (3 marks)

b) Given the following information about good x and y:

$$Qdx = 20 - 3Px + 4Py + 0.2Y$$

$$Px = 1 \quad Py = 2 \quad Y = 500$$

Where: Y = Consumer's Income

i) What can you say about the nature of good x? Prove your answer. (4 marks)

ii) What can you say about this statement: 'Good x and y are not related'. Prove your answer. (4 marks)

c) Using matrix algebra, determine the equilibrium values of interest rates and national income in the macro economy using the following IS and LM functions. (6 marks)

IS equation: $r = 8 - 0.006y$

LM equation: $r = -3 + 0.003y$