MACHAKOS UNIVERSITY
University Examinations for 2022/2023 Academic Year

SCHOOL OF BUSINESS, ECONOMICS AND HOSPITALITY AND TOURISM

MANAGEMENT

DEPARTMENT OF ECONOMICS

…………….. YEAR FIRST SEMESTER EXAMINATION FOR

MASTER OF ECONOMICS

EES 800: QUANTITATIVE METHODS FOR ECONOMISTS

DATE: **TIME:**

INSTRUCTIONS:

(i)   **Answer question ONE and any other THREE questions**

(ii)   **Show your workings clearly**

QUESTION ONE (COMPULSORY) (24 MARKS)

a)   Given the following   (12 marks)

$$Max \ F(x,y) = 2x^2y^2$$
$$subject \ to$$
$$18x + 3y \leq 1266$$
$$2x + 5y \leq 1000$$
$$x, y \geq 0$$

(i)   Use the Kuhn Tucker conditions to solve the following problem.   (6 marks)

(ii)   Construct the relevant Hessian determinant of this problem and evaluate it at the point obtained in part (a) to confirm that this is indeed a maximum.   (6 marks)

b)   Consider the market market model

$$Q_d = \alpha - \beta P_t \ldots \ldots \ldots \ldots (demand)$$
$$Q_s = -\gamma - \delta P_t \ldots \ldots \ldots \ldots (supply)$$
$$P_{t+1} = P_t - \sigma(Q_s - Q_d) \ldots \ldots \ldots \ldots (price \ adjustment)$$

Derive the time path P_t (6 marks)

c)   Give an exhaustive analysis of all possible time paths and how they arise.   (6 marks)

QUESTION TWO (12 MARKS)

Let the market demand for homogenous commodity be given by
$$P(Q) = a - Q$$

And firm costs be

$$C(q_i) = cq_i$$

Where c is constant for all firms

a) Construct a Cournot game for this situation in which there are two firms and solve it for industry output, price and profits. (4 marks)

b) What will be the industry results (output, price and profit be), if there was only one firm (a monopolist)? (4 marks)

c) Construct the alternative Betrand duopoly model and compare and contrast market results against those in the Cournot game. (4 marks)

QUESTION THREE (12 MARKS)

Consider the per unit input requirements in inter-industry shown in the matrix

$$A = \begin{bmatrix} 0.3 & 0.6 & 0 \\ 0.1 & 0.2 & 0.3 \\ 0.1 & 0.1 & 0.6 \end{bmatrix}$$

a) Derive the gross output vector X if the final demand vector D is given as (6 marks)

$$D = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$

Suppose the labour wage rate is w and the labour requirements per unit each good are

$$L = \begin{bmatrix} l_1 \\ l_2 \\ l_3 \end{bmatrix}$$

In addition, suppose the price of each good are given by

$$P = \begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix}$$

b) Compute the net income wage bill and profits of the entire economy. (6 marks)

QUESTION THREE (12 MARKS)

a) Write a first order linear difference equation (FODE) and use this to derive the general solution for this class of equation. Specify the particular solution and the complementary function. (6 marks)

b) Provide an exhaustive analysis of all possible alternative dynamic paths available for the complementary functions indicating the parameter values that determine ach path (4 marks)

c) Elaborate the application of the results from (b) in analysis of price adjustments in the demand and supply model. (2 marks)

QUESTION FOUR (12 MARKS)

A consumer has the following utility function $U = \alpha \ln X_1 + \beta \ln X_2$

Where X_1 is an ordinary consumption good per day and X_2 is the number of leisure hours per day (i.e $t = 24 - X_2$) is the number of working hours per day

Let P_1 be the price of X_1 and P_2 be the wage rate/price of leisure hours

Set out the constrained utility maximization problem clearly indicating what the constraint is
Construct the relevant Lagrangean and solve it to derive the deman functions X_1 and X_2 .

Derive the relevant Hessian determinant of this problem and establish the required results for a maximum
Analyse the effects of increase in the wage rate on the optimum results as completely as possible

QUESTION FIVE (12 MARKS)

a) Set up a first order linear differential equation (FOLDE) to illustrate the deviation of the general solutions of this class of equations showing the parameter restrictions needed for dynamic stability of the time path indicated by the solution. (6 marks)

b)   Let the demand and supply be: (6 marks)

* $Q_d = \alpha - \beta P + \sigma \frac{\partial p}{\partial t}$

* $Q_s = -\gamma + \delta P$

   $(\alpha, \beta, \gamma, \delta > 0$

Suppose the rate of change of the price over time is directly proportional to the excess demand. Derive the time path of the price and the market clearing price.