MACHAKOS UNIVERSITY

University Examinations 2022/2023

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF PHYSICAL SCIENCES

SECOND YEAR FIRST SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (TELECOMMUNICATION AND INFORMATION TECHNOLOGY)

SPH 205: MATHEMATICAL PHYSICS

DATE: **TIME:**

INSTRUCTIONS:

- The paper consists of **two** sections.
- Section **A** is **compulsory** (30 marks).
- Answer any **two** questions from section **B** (each 20 marks).

SECTION A

QUESTION ONE (COMPULSORY) (30 MARKS)

a) Find a vector 5 units long in the direction opposition to the direct of A= 2**i** –3**j**+6**k** (3 marks)

b) Find the angle between the vectors **A** = 2**i** –2**j** +**k** and **B** = 3**i** +4**k** (3 marks)

c) Find the gradient of the scalar field $\varphi = xy^2z^3 + x^3y$ (2 marks)

d) How much work does it take to slide a crate 20 m along a loading dock by pulling on it with a 200N force at an angle of 30^0 from the horizontal? (3 marks)

e) Find the divergence of the vector field $\mathbf{a} = x^2y^2\,\mathbf{i} + y^2z^2\,\mathbf{j} + x^2z^2\,\mathbf{k}$. (2 marks)

f) Parameterize the line segment joining the points P(-3, 2, -3) and Q (1, -1, 4) (3 marks)

g) Find an expression for the Laplace transform of $t\frac{d^2f}{dt^2}$. (5 marks)

h) The position vector of a particle at time t in Cartesian coordinates is given by $r(t) = 2t^2\mathbf{i} + (3t - 2)\mathbf{j} + (3t^2 - 1)\mathbf{k}$. Find the speed of the particle at t = 2 and the component of its acceleration in the direction $s = 2\mathbf{i} + \mathbf{j} + 2\mathbf{k}$. (4 marks)

i) A small particle of mass m orbits a much larger mass M centered at the origin O. According to Newton's law of gravitation, the position vector r of the small mass obeys the differential equation $m\frac{d^2\boldsymbol{r}}{dt^2} = -\frac{GMm}{r^2}\hat{\boldsymbol{r}}$. Show that the vector $\boldsymbol{r} \times \frac{d\boldsymbol{r}}{dt}$ is a constant of the motion. (5 marks)

SECTION B: ANSWER ANY OTHER TWO QUESTIOINS

QUESTION TWO (20 MARKS)

a) Express $2\mathbf{i} + \mathbf{j} + 2\mathbf{k}$ as a product of its length and direction (2 marks)

b) Find the curl of the vector field $\mathbf{a} = x^2y^2z^3\,\mathbf{i} + y^3z^2\,\mathbf{j} + x^4z^2\,\mathbf{k}$. (3 marks)

c) Show that $\nabla \cdot (\nabla\varphi \times \nabla\psi) = 0$, where φ and ψ are scalar fields. (5 marks)

d) A particle of mass m with position vector r relative to some origin O experiences a force **F**, which produces a torque (moment) $\mathbf{T} = \mathbf{r} \times \mathbf{F}$ about O. The angular momentum of the particle about O is given by $\mathbf{L} = \mathbf{r} \times m\mathbf{v}$, where **v** is the particle's velocity. Show that the rate of change of angular momentum is equal to the applied torque. (5 marks)

e) Given that $\mathbf{A} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$ and $\mathbf{B} = b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k}$ show that $\mathbf{A}.\mathbf{B} = a_1b_1 + a_2b_2 + a_3b_3$ using the cosine rule. (5 marks)

QUESTION TWO (20 MARKS)

a) Show that $\{\boldsymbol{e}_i\}$ and $\{\epsilon_j\}$ are reciprocal systems of vectors. (6 marks)

b) Find the distance from the point S (2,4,1) to the line (5 marks)

L: x = 1 + t   y = 3 - t   z = 2t

c) Evaluate the line integral $I = \int_C \boldsymbol{a} \cdot d\boldsymbol{r}$, where $\mathbf{a} = (x + y)\mathbf{i} + (y - x)\mathbf{j}$, along each of the paths in the xy-plane, namely the;

(i) Parabola $y^2 = x$ from (1, 1) to (4, 2), (3 marks)

(ii) Curve $x = 2u^2 + u + 1$, $y = 1+u^2$ from (1, 1) to (4, 2), (3 marks)

(iii) Line y = 1 from (1, 1) to (4, 1), followed by the line x = 4 from (4, 1) to (4, 2). (3 marks)

QUESTION FOUR (20 MARKS)

a) Using the Green's theorem, show that the area of a region R enclosed by a simple closed curve C is given by $A = \frac{1}{2}\oint_c (xdy - y\,dx) = \oint_c xdy = -\oint_c ydx$. Hence calculate the area of the ellipse $x = a\,cos\varphi, y = b\,sin\varphi$. (5 marks)

b) Find the total derivative of $f(x,y) = x^2 + 3xy$ with respect to x, given that $y = \sin^{-1} x$. (5 marks)

c) Prove the Rodrigues' formula $H_n(x) = (-1)^n e^{x^2} \frac{d^n}{dx^n}\left(e^{-x^2}\right)$ for the Hermite polynomials. (5 marks)

d) Find the general solution of the Bessel's equation given by

$x^2y'' + xy' + \left(x^2 - \frac{1}{4}\right)y = 0$ (5 marks)

QUESTION FIVE (20 MARKS)

a) Form a partial differential equation by eliminating the function φ from (5 marks)

$$lx + my + nz = \varphi(x^2 + y^2 + z^2)$$

b) Solve the differential equation $\frac{\partial^2 u}{\partial x \partial y} = \sin(x + t)$ given that $\frac{\partial u}{\partial x} = 1$ when $t = 0$ and when $u = 2t, x = 0$. (5 marks)

c) Find the volume of a box (parallelepiped) determined by **A** = **i** + 2**j** – **k** , **B = -2i + 3k** and **C = 7j - 4k** (4 marks)

d) The position vector of a particle in plane polar coordinates is $\boldsymbol{r}(t) = \rho(t)\hat{\boldsymbol{e}}_\rho$. Find expressions for the velocity and acceleration of the particle in these coordinates. (6 marks)