MACHAKOS UNIVERSITY

ISO 9001:2008 Certified

UNIVERSITY EXAMINATIONS 2016/2017

FIRST YEAR SECOND SESSION EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE EDUCATION

SMA 300: REAL ANALYSIS

DATE: …………………………….. TIME 2 HOURS

INSTRUCTIONS TO CANDIDATES

(a) Answer question one and ANY TWO Questions

QUESTION ONE 30 MARKS (COMPULSORY)

a) Define the following terms
   i) Set
   ii) Subsets complement
   iii) Universal set
   iv) Function
   v) Rational numbers (5 marks)

b) Show that $\sqrt{2}$ is irrational (4 marks)

c) The square of an odd integer is odd and the square of an even integer is even proof (4 marks)

d) Prove that the function $f(x) = x^2$ is continuous at every point a$\in R$ (4 marks)

e) Show that the series $\sum_{n=1}^{\infty}(-)^{n+1}\frac{1}{n^2}$ is convergent (3 marks)

f) Given that a and b are rational with b$\neq 0$ and s is an irrational number such that $a - bs = t$.
   Show that t is irrational hence show that $\frac{-1+\sqrt{3}}{1+\sqrt{3}}$ is irrational (4 marks)

g) i) Show that the infinite set $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4} \ldots\ldots\ldots\ldots\ldots\ldots. is\ bounded$ (3 marks)

ii) Determine the supremum and the infimum of the set in question g i) above (3 marks)

QUESTION TWO 20 MARKS

a) Let π be an arbitrary indexing set and G_α be a collection of open subsets of a metric space X . Prove that $\cup\, G_\alpha$ is also open in X. (5 marks)

b) Let (X, ∂) be a metric space and A $\subset X$. Then if P is a limit point of A. Prove that every nbd of P contains infinitely many points distinct from P. (5 marks)

c) Let (X, ∂) be a metric space. Prove that for any subset A of X with a finite number of elements cannot have limit point. (5 marks)

d) Prove that the intersection of an arbitrary family of closed sets is closed and the union of a finite number of closed sets is closed. (5 marks)

QUESTION THREE 20 MARKS

a) Show from the first principle that

i) $\lim\limits_{n\to\infty} \frac{1}{n}$=0

ii) $\lim\limits_{n\to\infty} \frac{3n+2}{n+1}$=3 (5 marks each)

b) Show that the sequence $\frac{1}{2^n}$ in R is a Cauchy sequence. (5 marks)

c) Proof that $\sqrt{3}$ is irrational (5 marks)

QUESTION FOUR 20 MARKS

a) If $\{A^n\}$is a collection of countable sets , then $A = \cup\, A^n$ is countable (5 marks)

b) Prove that a subset of a countable set is countable. (5 marks)

c) Prove that the empty set is open. (5 marks)

d) Prove that set of all even natural numbers is countable. (5 marks)

QUESTION FIVE 20 MARKS

a) Prove that the $f(x) = x^2 + 2x + 6$ is continuous at x=3 (5 marks)

b) Show that the function $f(x) = \begin{cases} \frac{3x^2-2x-8}{x-2} & , x \neq 2 \\ 8 & x=2 \end{cases}$

Is discontinuous at x=2.redefine the function to make it continuous at x=2 (5 marks)

c) Show that if $r = \sqrt{n+1}$-$\sqrt{n-1}$for any integer n>1 then r is rational (5 marks)

d) Consider the series $\sum_{n=1}^{\infty} \frac{1}{n^2+1}$,determine whether the series converges or diverges using the integral test. (5 marks)