On Changepoint Detection in a Series of Stimulus-Response Data

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January 30, 2017

Abstract

In this paper, we demonstrate the power of functional data models for a statistical analysis of stimulus-response experiments which is a quite natural way to look at this kind of data and which makes use of the full information available. In particular, we focus on the detection of a change in the mean of the response in a series of stimulus-response curves where we also take into account dependence in time.

Keywords: stimulus-response data, functional data, functional time series, changepoint test, inhibitory synaptic transmission

1 Introduction

Stimulus-response data are a frequent product of cognitive experiments. The test object is confronted with a stimulus, and the following response is measured in some form, e.g. as the changes in time of the potential at certain locations in a single neuron or by means of an electroencephalogram (EEG) of an animal or human. The full data are functions of time or, in the EEG case, vectors of functions. Usually, they are already digitized during storage, but with such a fine discretization such that they still can be seen as continuous curves.

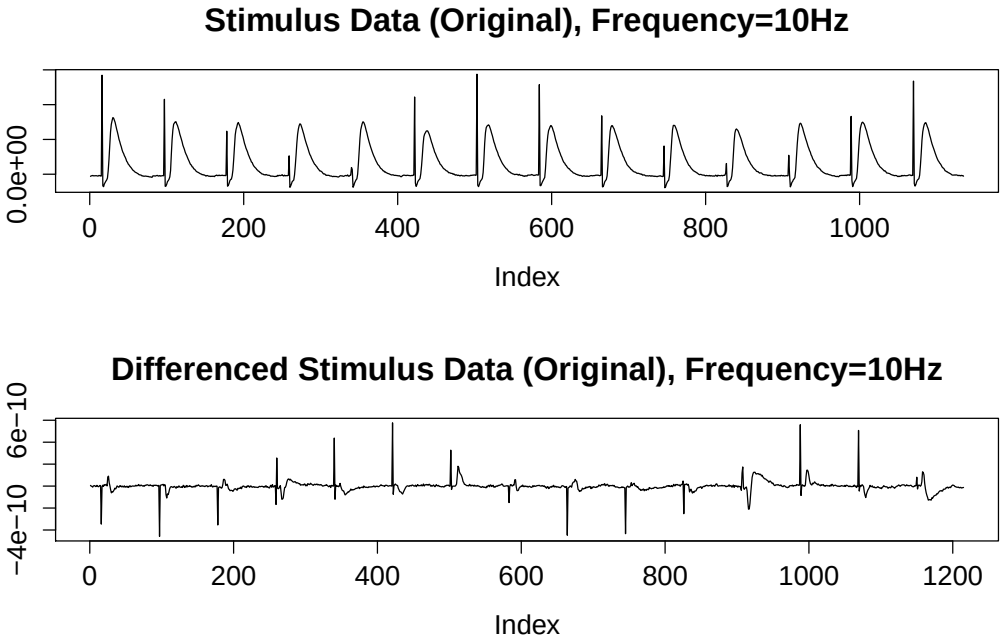
For a statistical analysis, we have to model such data as random functions of time. However, in cognitive science, the full information available is rarely used for inference. Usually, the response curves are reduced to a low-dimensional data vector before, e.g., performing statistical tests. Those data vectors consist of simple univariate characteristics like maximal response, average response, length of response, response latency, i.e. waiting time between stimulus and response etc. Modern functional data analysis allows to use the full information of the response curves in a quite natural manner which we want to demonstrate in this paper with a real-data example.

Standardizing the observation interval to $[0, 1]$, let $X_i(t), 0 \leq t \leq 1$, denote the response curve from the i^{th} experiment. Analogously to multivariate data, the mean curve of functional data is defined pointwise, i.e. $\mathrm{E}X_i(t) = \mu(t), 0 \leq t \leq 1$, if the functional data X_i have identical means. As for random vectors, there are tests for equality of the mean to some given function in case of one sample or for equality of the means of two independent samples (compare, e.g., **?**, chapter 5). In this paper, we consider a more involved testing problem. We have a time series of response curves $X_i(t), 0 \leq t \leq 1, i = 1, \ldots, N$, generated by presenting the same stimulus repeatedly to the same test object. The particular kind of data are explained in chapter 2.

In chapter 3, we consider the problem of testing for a change in the mean under the assumption of independent $X_1, \ldots, X_N$ as well as in the general setting of dependent curves. Such changepoints are of interest in experiments about learning or increasing fatigue of the test object under repeated stimuli. E.g., the response latency may become longer cor-

responding to a shift of the response curve towards the time of stimulus, or the response curves may become flatter on the average corresponding to the test object getting used to the stimulus. In chapter 4, we finally apply the methods described in chapter 3 to our actual stimulus response data and detect various changes in the mean in our sequences of stimulus-response curves. We also test the detrended data for dependence. It turns out that subsequent curves are dependent which has to be taken into account in the tests for changes in the mean.

Figure 1: Original stimulus-response data



2 Preprocessing the data

The data are generated by stimulus-response experiments on a single on a single neuron in the lateral superior olive, as part of a larger research project on the reliability of inhibitory synaptic transmission in the auditory brainstem. For more details about the physiological background, we refer to **?** or **?**. The stimulus is a brief electric shock that triggers synaptic activity and is repeatedly applied at various frequencies (1, 2, 5, 10, 50 Hz). The duration of the experiment is always 1 min such that the sample sizes for the samples with different

stimulus frequencies vary between $N = 60$ for 1 Hz and $N = 3000$ for 50 Hz. The individual responses are short-lived enough such that each response has ended well before the next stimulus even in case of the highest stimulus frequency. Hence, we have a series of curve data which look similar, but show some random variation.

The top panel of Figure **??** shows a subsection of 14 curve data from the experiment with stimulus frequency 10 Hz (observations number 11-24), where the total sample size was $N = 600$. Note that the horizontal axis shows the index number of discretized single measurements recorded for storage, not some physical time. We always stored about 85 observations for each individual stimulus-response cycle independently of the frequency.

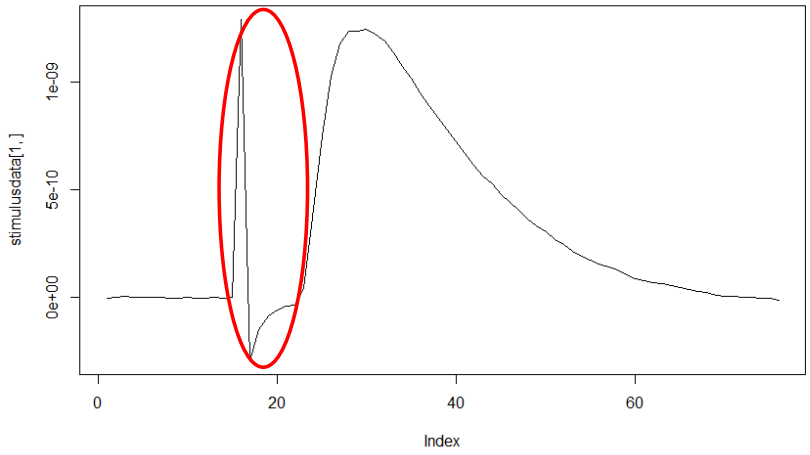
For the mean tests, we use the response curves themselves. In testing for dependence of the data, it is however convenient to first apply a differencing filter which removes the mean even in situations where it is slowly changing. To be precise, if $X_i(t)$ denote the original response curves, then the differenced curve data are the random functions

$$Y_i(t) = X_{i+1}(t) - X_i(t), \quad i = 1, \ldots, N-1. \tag{1}$$

The lower panel of Figure **??** shows a subsection of the differenced response curves from the experiment with stimulus frequency 10 Hz.

At the beginning of each response and differenced response there is a noticeable sharp spike (circled in red) in Figure **??**. This is an artifact which represents the direct effect of the stimulus onto the measuring device, but does not correspond to the response of the cell. The cell reacts to the stimulus only after a brief delay. As the stimulus part and the response part of the curves are well enough separated and we are only interested in the measurements of the response, it is safe to remove a few data points at the beginning of each curve. We therefore cut the data points in the circle and consider only the rest as the reponse curve to be analyzed further on. Once the truncation has been done, we have 68, 73, 78, 73, 73 measurement points per individual curve left in the case of 1, 2, 5, 10 and 50 Hz frequencies respectively, which are then smoothed to form the curves shown in the figures.

Figure 2: Artifact



Figures **??**, **??** and **??** show the adjusted and differenced plots of parts of the response curve samples corresponding to stimulus frequencies 1, 2, 5, 10 and 50 Hz respectively. In particular, after the adjustment the local random noise in the differenced data can be seen much more clearly.

3 Testing for changes in the mean

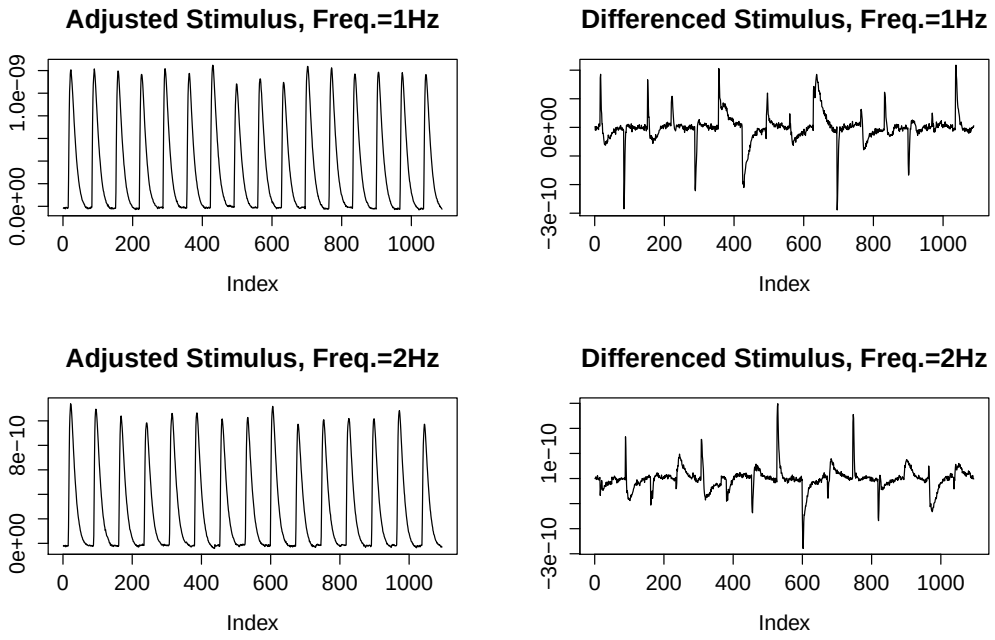
We interpret observed response curves resp. their transformations after preprocessing as random functions $X_i(t), 0 \leq t \leq 1$, and we assume that they are square integrable:

$$\int_0^1 X_i^2(t)dt < \infty,$$

i.e. X_i is a random variable with values in the space $\mathcal{H} = L^2[0,1]$ of, for convenience complex-valued, square integrable functions on [0,1]. This space is a separable Hilbert space which has a quite similar structure as the finite dimensional Euclidean space $\mathbb{R}^m$. In particular, there is a scalar product and a corresponding norm

$$\langle f, g\rangle = \int_0^1 f(t)\overline{g(t)}dt, \qquad \|f\| = \left\{\int_0^1 |f(t)|^2 dt\right\}^{1/2}, \qquad f, g \in \mathcal{H},$$

Figure 3: Adjusted Responses (left) and their Differenced Counterparts 1, 2 Hz



where $\overline{g(t)}$ denotes the complex conjugate of $g(t)$. There exists a countable orthonormal basis, i.e. a sequence of functions $v_1, v_2, \ldots$ in $\mathcal{H}$ with $\|v_k\| = 1$, $\langle v_k, v_l \rangle = 0$ for all $k \neq l$, such that we have the usual linear expansion of any f in $\mathcal{H}$ in terms of the basis

$$f(t) = \sum_{k=1}^{\infty} \langle f, v_k \rangle v_k(t), \qquad \|f\|^2 = \sum_{k=1}^{\infty} \langle f, v_k \rangle^2.$$

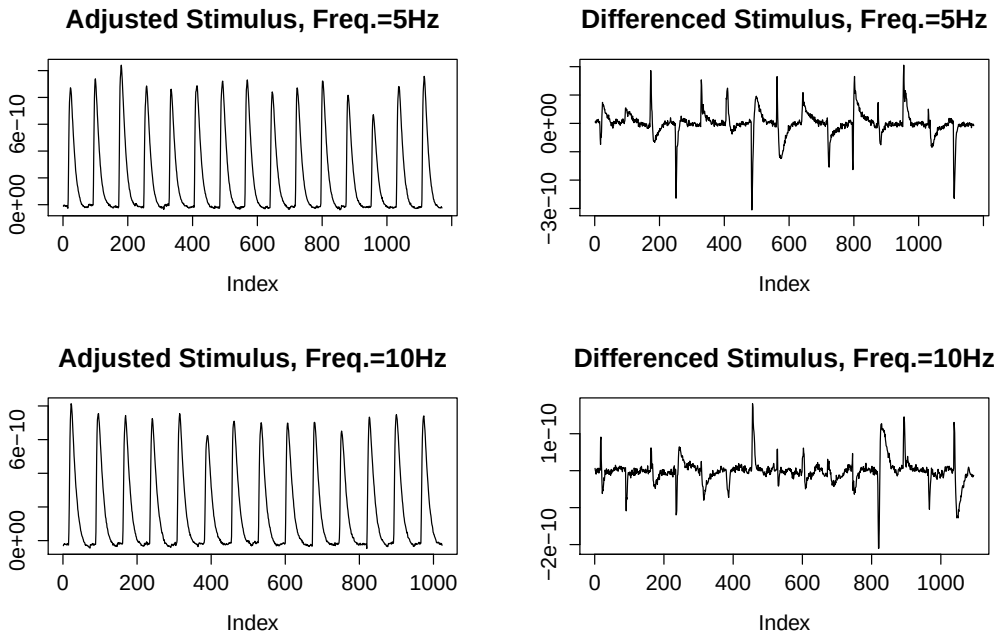
If we choose, in particular, the Fourier basis $v_k(t) = e^{i2\pi kt} = \cos(2\pi kt) + i \sin(2\pi kt)$, $-\infty < k < \infty$, then this is the Fourier expansion well known in signal analysis, and $\langle f, v_k \rangle$ are the Fourier coefficients of f. In the following, we refer some notions and results from chapter 6 of **?**.

3.1 Changepoint test for independent data

If $X_i(t), i = 1, \ldots, N$, is a sequence of real-valued random functions in $\mathcal{H}$, then we decompose them into the mean function and the random component:

$$X_i(t) = \mu_i(t) + Y_i(t), \qquad \mathrm{E}Y_i(t) = 0.$$

Figure 4: Adjusted responses (left) and their differenced counterparts 5, 10 Hz



We assume that the random components Y_i are independent and all have the same distribution satisfying

$$\mathrm{E}\|Y_i\|^2 = \int_0^1 Y_i^2(t)dt < \infty.$$

Then, the covariance function measuring dependence between the function values $X_i(t), X_i(s)$ at different points t, s in time, does not depend on i:

$$c(t,s) = \mathrm{cov}(X_i(t), X_i(s)) = \mathrm{E}Y_i(t)Y_i(s) \qquad \text{for all } i, 0 \le s, t \le 1,$$

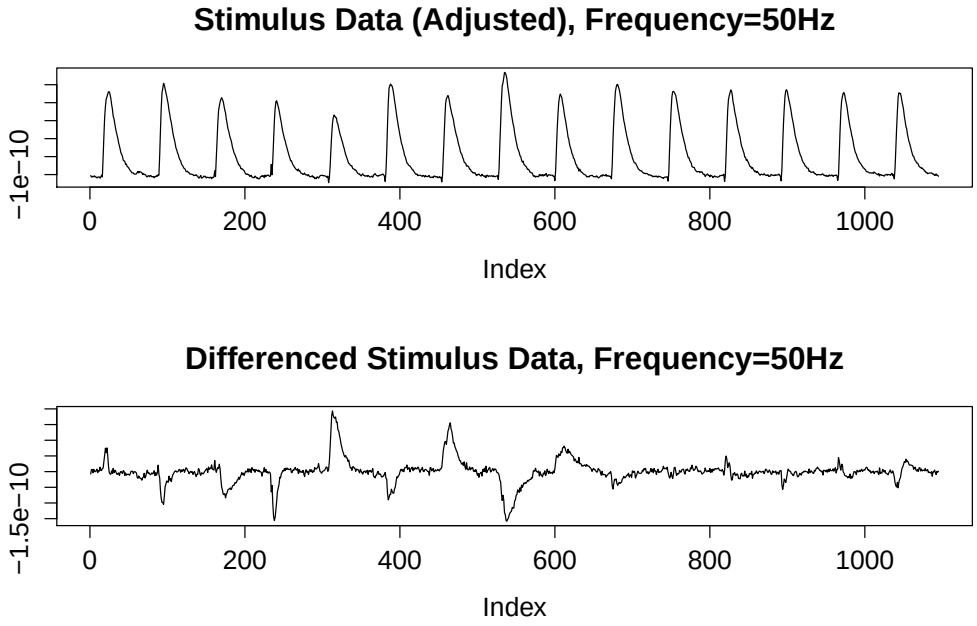
and it allows for the expansion

$$c(t,s) = \sum_{k=1}^{\infty} \lambda_k v_k(t) v_k(s).$$

$\lambda_1 \ge \lambda_2 \ge \ldots$ are the ordered eigenvalues, which automatically are nonnegative, and $v_1, v_2, \ldots$ the corresponding orthonormal eigenfunctions of the covariance operator C which linearly maps a function f in $\mathcal{H}$ onto the function Cf given by

$$(Cf)(t) = \mathrm{E}\big(\langle Y_i, f\rangle Y_i(t)\big) = \mathrm{E}\big(\int_0^1 Y_i(s)f(s)ds Y_i(t)\big) = \int_0^1 c(t,s)f(s)ds. \qquad (2)$$

Figure 5: Adjusted responses (top) and their differenced counterparts 50 Hz



The functions $v_1, v_2, \ldots$ are called the functional principal components. As they are an orthonormal basis of $\mathcal{H}$, we also have

$$Y_i(t) = \sum_{k=1}^{\infty} y_{i,k} v_k(t), \qquad \text{where } \; y_{i,k} = \langle Y_i, v_k \rangle.$$

We want to test if the response curves are on the average identical or, if at some unknown changepoint m in the sample, the mean changes. In our model above, the null hypothesis H_0 of no change and the alternative H_1 of one change are

$$\mathrm{H}_0 : \mu_1 = \ldots = \mu_N, \qquad \mathrm{H}_1 : \mu_1 = \ldots = \mu_m \neq \mu_{m+1} = \ldots = \mu_N \;\text{ for some } 1 \leq m < N.$$

As the basis of the test statistic, we consider the partial means of data before and after k:

$$\hat{\mu}_k(t) = \frac{1}{k} \sum_{i=1}^{k} X_i(t), \qquad \tilde{\mu}_k(t) = \frac{1}{N-k} \sum_{i=k+1}^{N} X_i(t).$$

Under H_0, both $\hat{\mu}_k$ and $\tilde{\mu}_k$ will estimate the common mean of all the functional data and will be approximately equal for all k. If, however, there is a changepoint $m < N$, then $\hat{\mu}_k - \tilde{\mu}_k$ will be large for $k \approx m$.

For small k, the variability of $\hat{\mu}_k$ is rather large, as only few observations contribute to the average, and the same applies to $\tilde{\mu}_k$ for small $N-k$. Therefore, the test uses the weighted differences

$$P_k(t) = \frac{k(N-k)}{N}\left(\hat{\mu}_k(t) - \tilde{\mu}_k(t)\right) = \sum_{i=1}^{k} X_i(t) - \frac{k}{N}\sum_{i=1}^{N} X_i(t),$$

to take into account the different random variability of $\hat{\mu}_k - \tilde{\mu}_k$ for various k.

If P_k would be scalar numbers, we would look at the maximum value of $|P_k|$ in the spirit of classical changepoint analysis and reject the hypothesis H_0 if it exceeds a critical bound depending on the level of the test. However, P_k is a function in $\mathcal{H}$. We could reduce them to scalar characteristics like the integral of the absolute value or the maximum if we would have a rather precise notion about the type of change to expect. A main feature of functional data analysis, however, is its flexibility regarding the characterization of response curves. So, we are looking for several scalar quantities which combined give us the essential features of the whole function. For a suitable d (compare subsection **??**), these are just the scores of P_k relative to the first d functional principal components $v_1, \ldots, v_d$, i.e.

$$\langle P_k, v_\ell \rangle = \int_0^1 P_k(t) v_\ell(t) dt, \ \ell = 1, \ldots, d.$$

Then, for convenience, we look at a suitable weighted average of the squares, not of the absolute values, of the $\langle P_k, v_\ell \rangle$:

$$T_N(k) = \frac{1}{N}\sum_{\ell=1}^{d} \frac{1}{\lambda_\ell} \langle P_k, v_\ell \rangle^2.$$

This is not yet a feasible test statistics, as it depends on the unknown v_ℓ, λ_ℓ. First note that

$$\langle P_k, v_\ell \rangle = \langle \sum_{i=1}^{k} X_i - \frac{k}{N}\sum_{i=1}^{N} X_i, v_\ell \rangle = \langle \sum_{i=1}^{k} Y_i - \frac{k}{N}\sum_{i=1}^{N} Y_i, v_\ell \rangle = \sum_{i=1}^{k} y_{i,\ell} - \frac{k}{N}\sum_{i=1}^{N} y_{i,\ell},$$

as centering each summand in both sums by subtracting $\overline{X}_N$ has no effect. Therefore, for estimating $T_N(k)$, we need to estimate $\lambda_\ell, y_{i,\ell}, \ell = 1, \ldots, d, i = 1, \ldots, N$. First we estimate the covariance function $c(t,s)$ by the sample version

$$\hat{c}(t,s) = \frac{1}{N}\sum_{i=1}^{N} \big(X_i(t) - \overline{X}_N(t)\big)\big(X_i(s) - \overline{X}_N(s)\big)$$

where, under the hypothesis of no change, the sample mean $\overline{X}_N(t)$ of $X_1(t), \ldots, X_N(t)$ estimates the common mean function of the curve data. $\hat{c}(t,s)$ characterizes the estimate $\widehat{C}$ of the covariance operator analogously to (**??**). Finally, we have to calculate the first d eigenvalues $\hat{\lambda}_1 > \ldots > \hat{\lambda}_d$ and the scalar products of the centered data with the corresponding eigenvectors $\hat{v}_1, \ldots, \hat{v}_d$ of $\widehat{C}$ to get the estimate of $T_N(k)$

$$\widehat{T}_N(k) = \frac{1}{N} \sum_{\ell=1}^{d} \frac{1}{\hat{\lambda}_\ell} \Big(\sum_{i=1}^{k} \hat{y}_{i,\ell} - \frac{k}{N} \sum_{i=1}^{N} \hat{y}_{i,\ell} \Big)^2 .$$

These calculations can be easily done using the **R** package fda. There are various possibilities how to combine $\widehat{T}_N(k), k = 1, \ldots, N$ to a single scalar test statistic. **?** just use averaging and get

$$S_{N,d} = \frac{1}{N} \sum_{k=1}^{N} \widehat{T}_N(k) = \frac{1}{N^2} \sum_{\ell=1}^{d} \frac{1}{\hat{\lambda}_\ell} \sum_{k=1}^{N} \Big(\sum_{i=1}^{k} \hat{y}_{i,\ell} - \frac{k}{N} \sum_{i=1}^{N} \hat{y}_{i,\ell} \Big)^2 .$$

H_0 is rejected if $S_{N,d}$ is large. Let us just summarize again the intuition behind this decision procedure. As mentioned above, if the mean does not change, the weighted differences $P_k(t)$ of the sample mean functions before and after k should all be reasonably close to 0. Hence, for all k and ℓ, their squared scores $\langle P_k(t), v_\ell \rangle^2$ should be small. Now, $T_N(k)$ as a weighted average of those quantities should be small too for $k = 1, \ldots, N$, and, hence, this also holds for the average over k. If we replace the unknown quantities in this average by their sample analogues, then we just get $S_{N,d}$.

Finally, we need critical values for the test which we get from the asymptotic distribution of $S_{N,d}$ under the hypothesis which has been derived be **?** under some rather weak regularity assumptions. In particular, for $N \to \infty$

$$\text{pr } (S_{N,d} > z \mid \mathrm{H}_0 \text{ holds }) \to K_d = \int_0^1 \sum_{\ell=1}^{d} B_\ell^2(t) dt, \tag{3}$$

where $B_\ell, \ell = 1, \ldots, d$, are independent standard Brownian bridges. The distribution of K_d has been derived quite early by **?** in his study of extensions of the Cramér-von Mises test. Critical values for $S_{N,d}$ for various significance levels and values of d can be found in Table 6.1 of **?**.

If the test rejects the hypothesis and detects a changepoint m, then we are interested in estimating its location. A consistent estimate $\hat{m}$ is derived by checking at which index k, the statistic $\widehat{T}_N(k)$ assumes its maximum:

$$\widehat{T}_N(\hat{m}) = \max_{k=1,\ldots,N} \widehat{T}_N(k). \tag{4}$$

Note that if we detect a changepoint, we can say that the mean is not constant over time, i.e. H_0 does not hold, up to the usual small error probability. It does not necessarily imply that the mean is constant before and after the changepoint. The test is also sensitive against other kinds of alternatives, e.g. several changepoints or a gradual change of the mean.

One way to check the constancy of the mean before and after the detected changepoint is a repeated application of the test. So, if H_0 is detected and $\hat{m}$ is the estimated changepoint, we apply the test again twice to the samples $X_1, \ldots, X_{\hat{m}}$ resp. $X_{\hat{m}+1}, \ldots, X_N$. If we detect some changepoints in those subsamples, then again we split the samples and apply the test again until finally we get a partition of the original data into subsamples which all have approximately constant means or just have small enough sample sizes that the test does not reject the hypothesis any longer.

3.2 Changepoint test for dependent data

We now consider the same setting as in the previous subsection, but we allow for dependence of the curve data. In particular, we assume that the random functions $Y_1, \ldots, Y_N$ centered around 0 are part of a stationary times series of functional data which satisfies certain weak dependence conditions (compare chapter 16 of **?**). We again want to test for a change in the mean. The testing procedure is similar, but, as in the familiar scalar setting, we have to take into account that the variability of the sample mean $\overline{X}_N(t)$ depends on the kind of dependence of the data. In particular, the variability will be larger if the dependence is rather positive which is the more common situation in practice. This would lead to a larger number of false rejections of the above test procedure if we falsely assume

independence. Therefore, we have to modify the test statistics accordingly. We follow the work of **?**, also described in **?**.

As in the scalar case, the effect of dependence on mean tests can be summarized in the long-run variance. For a real-valued stationary time series $Z_t, -\infty < t < \infty$, with mean 0 this quantity is the sum over all autocovariances

$$\sigma = \sum_{h=-\infty}^{\infty} \mathrm{cov}(Z_t, Z_{t+h}) = \sum_{h=-\infty}^{\infty} \mathrm{E} Z_t Z_{t+h}.$$

By stationarity, it does not depend on t. Equivalently, σ is the value of the power spectral density of the time series at 0.

The functional data enter the test statistic of the previous subsection only in form of the scores $\hat{\mathbf{y}}_i = (\hat{y}_{i,1}, \ldots, \hat{y}_{i,d})^{\mathrm{T}}, i = 1, \ldots, N$, which is a sequence of d-dimensional random vectors. So, we need the long-run variance which now is a $d \times d$-covariance matrix, of a d-variate stationary time series $\mathbf{z}_t, -\infty < t < \infty$, with mean 0 which is defined as

$$\boldsymbol{\Sigma} = \sum_{h=-\infty}^{\infty} \mathrm{E} \mathbf{z}_t \mathbf{z}_{t+h}^{\mathrm{T}}.$$

To get an estimate of $\boldsymbol{\Sigma}$, we estimate the autocovariances $\Gamma_h = \mathrm{E} \mathbf{z}_t \mathbf{z}_{t+h}^{\mathrm{T}}$ by their empirical versions based on a sample $\mathbf{z}_1, \ldots, \mathbf{z}_N$:

$$\widehat{\Gamma}_h = \frac{1}{N} \sum_{i=1}^{N-h} \mathbf{z}_i \mathbf{z}_{i+h}^{\mathrm{T}}, \ 0 \le h < N, \qquad \widehat{\Gamma}_h = \widehat{\Gamma}_{-h}, \ -N < h < 0.$$

Then, we apply the windowing technique well known from one-dimensional spectral analysis to get with some suitable window width b_N depending on N

$$\widehat{\boldsymbol{\Sigma}}_N = \sum_{h=-N+1}^{N-1} K\big(\frac{h}{b_N}\big) \widehat{\Gamma}_h.$$

K is a common kernel function which is bounded, symmetric around 0 and, for convenience, has a bounded support, say $[-1, +1]$. An example is the Bartlett kernel $K(u) = 1 - |u|$ for $|u| \le 1$, and $K(u) = 0$, else. For $N, b_N \to \infty$ such that $b_N/N \to 0$, $\widehat{\boldsymbol{\Sigma}}_N$ is a consistent estimate of $\boldsymbol{\Sigma}$ under some regularity conditions.

For getting an appropriate test statistic, set for $1 \leq k \leq N$

$$L_N(k) = \frac{1}{N}\left(\sum_{i=1}^{k} \hat{\mathbf{y}}_i - \frac{k}{N}\sum_{i=1}^{N} \hat{\mathbf{y}}_i\right).$$

Let $\widehat{\mathbf{\Sigma}}_N(\hat{\mathbf{y}})$ denote the long-run variance estimate based on $\mathbf{z}_i = \hat{\mathbf{y}}_i, i = 1, \ldots, N$, and set

$$R_{N,d} = \frac{1}{N}\sum_{k=1}^{N} L_N^{\mathrm{T}}(k)\widehat{\mathbf{\Sigma}}_N^{-1}(\hat{\mathbf{y}})L_N^{\mathrm{T}}(k).$$

Note that for the diagonal matrix with entries $\hat{\lambda}_1, \ldots, \hat{\lambda}_d$ replacing $\widehat{\mathbf{\Sigma}}_N(\hat{\mathbf{y}})$, the integrand coincides with $\widehat{T}_N(k)$ such that $R_{N,d}$ is a straightforward generalization of the test statistic $S_{N,d}$ of the previous section to the dependent case. The asymptotics does not change under the hypothesis and under the alternative as the effects of dependence are completely covered by the modification of the test statistic. Therefore, we may use the critical values from Table 6.1 of **?** for the changepoint test under dependence, too.

Note that in chapter 16 of **?** a slightly different version of the test statistic is considered, but it differs from ours only by replacing an integral by the corresponding Riemann sum which asymptotically is neglible.

Figure 6: Scree Plot

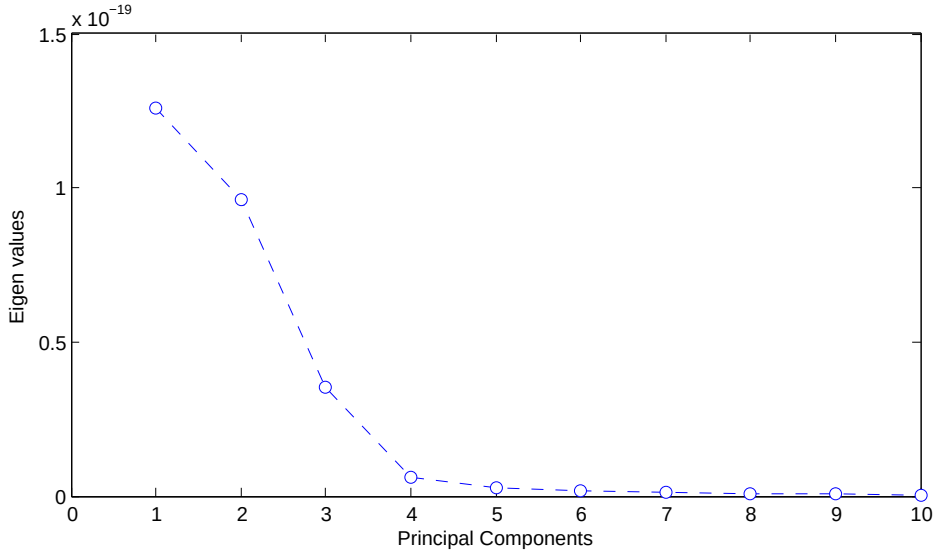


Table 1: Test for change in the mean function (i.i.d. Test)

$\alpha = 0.05$, d=4, Asymptotic crit. value=1.239675					
	1Hz	2Hz	5Hz	10Hz	50Hz
N	60	120	300	600	3000
Test statistic	2.0872	3.8249	8.5994	54.7244	212.0775

4 Application to Stimulus Response Data

Before applying the changepoint test, we have to choose the number d of functional principal components entering the test statistic. This problem is closely related to to the analogous problem in classical principal component analysis as a tool for dimension reduction, and we use a popular method, which is based on the scree plot, for selecting the number of relevant principal components based on the data.

The screeplot shows how much each principal component contributes to the total variability of the data in decreasing order of importance. In the case of functional principal components, the contribution to total variability are just given by the eigenvalues $\lambda_1 \geq \lambda_2 \geq ...$ of the covariance operator C introduced in subsection **??**. Estimates $\hat{\lambda}_j$ are easily calculated using the fda package of **R**. Figure **??** shows the screeplot for the sample corresponding to the stimulus frequency 10 Hz. The screeplots of the other samples look quite similar.

The idea of the scree plot is that we visually select the number d of principal components as the point where the curve dies off. Another more objective method for this purpose is requiring that the cumulative percentage of variance explained by the first d functional principal component has to be greater than some bound close to 100%, e.g. 85%. Based on Figure **??** and this rule, we decided to work with $d = 4$ functional principal components. They explain a cumulative percentage of variance of approximately 96%.

Assuming the data is independent and identically distributed, we applied the test de-

Table 2: Changepoints in order of significance (i.i.d. Test)

Frequency	Change Points						
1Hz	20						
2Hz	70	100					
5Hz	155	85					
10Hz	361	164	62	10	472	396	547

scribed in subsection **??** to the data with stimulus frequency 1, 2, 5, 10 and 50 Hz. The data where adjusted to remove the artificial artifact, but not differenced. Table **??** reports the results obtained for significance level 0.05. Note that the asymptotic critical value, based on the relationship (**??**), does not depend on the sample size N due to an appropriate standardization of the test statistics $S_{N,d}$ such that it is the same for all stimulus frequencies. In all cases, a changepoint was detected as the values of the test statistic all exceed the critical value.

Once the changepoint was detected, we estimated it using (**??**). Then, we splitted the sample and applied the test repeatedly until no further changepoints were detected. In Table **??** we list the detected changepoints in order of significance for the frequencies 1, 2, 5 and 10 Hz. These will be used for comparison with the changepoints for the dependent case. The changepoints are listed here as number of observed stimulus-response curve in the sample and not as physical time.

We also carried out the test for a change in the mean using the differenced data. As expected, for all frequencies no change point was detected which implies that these data approximately have a constant mean.

As discussed in subsection **??**, the test of subsection **??**, which is based on the assumption of independence, is known to give wrong results when the data show some dependency, likely too many rejections of the hypothesis. As we suspected dependence in our data which are response curves measured subsequently on the same cell, we tested for dependence. We

Table 3: Portmanteau Test

$\alpha = 0.05$, d=4, Asymptotic crit. value=67.5050					
	1Hz	2Hz	5Hz	10Hz	50Hz
N	59	119	299	599	
Test statistic	176.3522	313.8736	334.5219	552.3081	2574.5181

carried out a Portmanteau test presented by **?** for testing the hypothesis H_0 of independence of the curve data $X_1, \ldots, X_N$ against an open ended alternative of lack of independence or of sameness of distributions. The corresponding test statistic is asymptotically chi-square distributed under the null hypothesis, such that critical values are well-known. The main assumption of the test is the existence of fourth moments of the observations which is likely be satisfied looking at the data. Also, the data should be stationary which of course is not true if the means are changing. Therefore, we applied the test to the differenced data Y_j given by (**??**). The results of the test are given in Table **??**. In all cases the assumption of independence is rejected such that our data are genuine functional time series.

As the data are likely dependent, the previous application of the test of subsection **??** is not justified. Therefore, we dropped the assumption of independence and applied the more complex test of **?** described in subsection **??**. The results of the tests are reported in Table **??**; in all case we again detect a change in the mean on the significance level 0.05. However, the values of the test statistics are generally smaller. As the asymptotic distribution of the statistics $S_{N,d}$ and $R_{N,d}$ are identical, this means that the hypothesis is not so strongly rejected as if we falsely use the test for independent data.

Note that, as under the incorrect assumption of i.i.d. curve data, the test taking into account dependence also accepts the hypothesis of no change for all stimulus frequencies if we apply it to the differenced data data Y_i.

The differences between the two tests of subsections **??** and **??** are more striking once we apply it repeatedly to the split subsamples in search of more than one changepoint.

Table 4: Test for change in the mean function (Dependent Test)

$\alpha = 0.05$, d=4, Asymptotic crit. value=1.239675						
	1Hz	2Hz	5Hz	10Hz	50Hz	
N	60	120	300	600	3000	6000
Test statistic	1.5847	2.0715	3.6859	8.6769	32.6208	

Table 5: Changepoints in order of significance (Dependent Test)

Frequency	Change Points								
1Hz	20								
2Hz	74								
5Hz	155								
10Hz	359	163	62	472	389				
50Hz	2067	1213	679	288	182	542	358	987	1081
	1787	1632	1924	2459	2330	2748	2591	2830	

Table **??** gives the change points in order of their significance based on the changepoint test for dependent data. Comparing the results to those from Table **??**, we see that the test of subsection **??** for i.i.d. data detects many false changepoints as a result of failure to account for the long-run variance. Also, it is noticeable as expected, that with increasing frequency of the stimulus there are more changepoints. This can be attributed to the fact that at high frequency the cell does not have enough time to recover and go back to its resting state before the next stimulus is given.

5 Summary

In this paper, we applied tests from functional data analysis to illustrate their merit in making use of the full information in stimulus response curve data. In particular, we showed that the subsequent detrended curve data are dependent. Using an appropriate changepoint test which takes into account the dependence, we were able to show that the original

curve data showed several changes in the mean response curve throughout the experiment.

Our findings are in accordance with other statistical analyses of the same data. E.g., looking only at the univariate response latencies, i.e. the time span between stimulus and start of the response, we found an increasing trend which also was not homogeneous but showed changepoints between periods of rapid increase and periods of almost constancy.

Acknowledgement: This paper was supported by the PhD programme *Mathematics in Industry and Commerce* (MIC), funded by the German academic exchange service (DAAD), and by the *Center for Cognitive Science*, Technische Universität Kaiserslautern

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