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Crank Nicolson Approach for the Valuation of the Barrier Options

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Abstract

Barrier options are considered to be cheaper than the standard options as they provide lower premiums which make them to be attractive to hedgers in the financial market. In this article, the numerical procedure for the valuation of the barrier option is presented under the Black-Scholes framework. This procedure incorporates the explicit together with implicit finite difference approach. To obtain an accurate price of the considered barrier options, a grid is constructed such that we have the barrier located in a suitable position. The numerical result obtained is being compared with the exact value of the option and it shows that this approach converges faster to the exact value with smaller time step.

AMS subject classification: 65Nxx.
Keywords: Barrier Options, Crank Nicolson, Option Pricing, Partial Differential Equations.

1. Introduction

Option pricing has arguably become one of the major areas in the modern financial theory and practice. Though the earlier form of the option are the European and American option in which the former allows the exercise of a right at the maturity time unlike the latter where the holder of the option can exercise the right at any desirable time during the life of the option. However, an option can either be a call or a put. A call option gives the holder the right to buy an underlying asset in which the writer is obligated to sell should the holder chose to exercise the right while the put option provide the holder the right to sell an underlying asset where the writer is obligated to buy should the holder exercise the right. Apart from the standard option, there are also several options whose behaviour differs from the ones described earlier and they are being refer to as the exotic option. The Barrier options belong to this class though its characteristics are somewhat similar to those of the standard option however, it provides additional advantages and as a result, risk managers consider it to be cheap and it gives the privilege to hedge against the possible risk exposure [1].

Pricing of the barrier option is dated back to 1973 when [2] presented a closed-form solution for the pricing of a continuously monitored down and out European call. Also there is a lattice pricing proposed by [3] where they devise a simplified approach for option pricing and this has been used by many to price several forms of exotic options including barrier options. Furthermore [4] extends their work on binomial model by proposing the trinomial tree model which have three states in the pricing of an option and this gives a better accuracy with lower time step compared to the binomial model. However, [5] pointed out that the CRR binomial model proposed in [3] and the Boyles trinomial model in [4] used in the valuation often lead to error and requires large number of time steps if applied to the barrier options. Similarly, [6] indicates that pricing and hedging of barrier options using a binomial lattice can be quite delicate and as a result, an efficient trinomial lattice procedure which can be used to price and hedge barrier options that guarantee a more accurate value is proposed. Apart from using the lattice tree model for the pricing and hedging of barrier options, finite difference approximation can also be used and this often gives a more accurate result when applied to the exotic options. This method was first used by [7] where they used the finite difference methods and jump processes arising in the pricing of contingent claims. They established that the explicit finite difference approximation coincide to a three-point jump process discussed by [8]. Furthermore, [5] modified the explicit finite difference method for the pricing of barrier option. They ensure that the boundary conditions at the barriers are properly set up before the arising partial differential equation is approximated numerically and their approach provides a simple and effective way of valuing the barrier options when the asset price is close to the barrier. However, the explicit method is numerical unstable unless a stability condition is attached and this often leads to high running time and more memory requirements. Therefore in order to find a remedy for this, the implementation of Crank Nicolson approach is inevitable.

This paper presented the Crank Nicolson finite difference approach for the valuation of the barrier options. The approach is intuitive which ensures that boundary conditions at the barrier are rightly set. The rest of the paper is organised as follows. Section 2 focuses on the barrier options where the knock in and out are considered together with their payoff. Section 3 presents the Crank Nicolson finite difference approach to the pricing of barrier options while the stability analysis is discussed in section 4. A numerical example is presented in section 5 and section 6 concludes the paper.

2. Barrier Options

Barrier options is regarded as one of the oldest forms of the exotic options though it has been traded sporadically in the U.S market since 1967, six years before the establishment of the Chicago Board of Option Exchange (CBOE) and these options were geared to the needs of sophisticated investors such as managers of hedge funds [1]. In general, barrier options were established to provide risk managers with cheaper returns to hedge their exposures.

The option is regarded to be conditional, which is highly dependent on whether some barriers are violated within the life of the options and they are therefore known as a path dependent. They can be classified into two: knock-in and knock-out. Knock-in options only exist if during the life of the option, the underlying asset hit a specified barrier. It is a class of barrier option in which the holder is entitled to receive a European option if the barrier is hit and a rebate or nothing at the expiration if otherwise. Knock-in options may also be subdivided into down (up). In the case of up and in option, the option exist if the asset price move above the defined barrier and the converse apply for the down and in.

Contrary to knock-in barrier options, a knock-out option enables the owner to receive a rebate when a barrier is hit and a European option if otherwise. During the life of the option, the knock-out barrier options stop existing when the underlying asset price hit the barrier.They can also be divided into up or down out option. The up and out does not exist if the underlying asset price moves beyond the barrier while for down and out, when the asset price moves below the barrier then the option do not exist [9]. If an underlying asset reaches the barrier at any time during the option's life, the option is knocked out, or terminated.

2.1. Terminal Payoff of Barrier Options

The payoff of the barrier options depend on whether one or more barriers are breached within the life of the options. Given the spot underlying asset price, the barrier B can be placed either above or below. K is the strike price of the of the European option and we denote the stock price at time t as S_t. Let $M_s = \max\{S_t,\ t \in (0, T)\}$ and $m_s = \min\{S_t,\ t \in (0, T)\}$ where T is the expiry time. The payoff of different barrier calls and put option is given by:

2.1.1 Down and in Call/put

The down and in call option gives the holder the right to receive a vanilla call option if the barrier (B) is reached or a rebate if otherwise. To have a down and in option, the barrier is set such that $m_s > B$ and if during the life of the option, the asset price $m_s \leq B$ then the option becomes active and the holder is eligible to receive the payoff of the vanilla call. The payoff of a down and in (DI) can be expressed as:

$$DI_{call} = \begin{cases} R, & m_s > B \\ [S_T - K]^+, & m_s \leq B \end{cases}$$

$$DI_{put} = \begin{cases} R, & m_s > B \\ [K - S_T]^+, & m_s \leq B \end{cases}$$

2.1.2 Up and in Call/put

Here, the barrier is set such that $M_s < B$ and if during the life of the option, the asset price $M_s \geq B$ then the option becomes active and the holder is eligible to receive the payoff of the vanilla call. The payoff of a up and in (UI) can therefore be expressed as:

$$UI_{call} = \begin{cases} R, & M_s \leq B \\ [S_T - K]^+, & M_s > B \end{cases}$$

$$UI_{put} = \begin{cases} R, & M_s < B \\ [K - S_T]^+, & M_s \geq B \end{cases}$$

2.1.3 Down and out Call/put

The down and out call option gives the holder the right to receive a vanilla call option if the barrier is not reached or a rebate if otherwise. To have a down and out option, the barrier is set such that $m_s > B$ and if during the life of the option, the asset price move such that $m_s \leq B$ then the option became inactive. The payoff of a down and out (DO) can be expressed as:

$$DO_{call} = \begin{cases} [S_T - K]^+, & m_s > B \\ R, & m_s \leq B \end{cases}$$

$$DI_{put} = \begin{cases} [K - S_T]^+, & m_s > B \\ R, & m_s \leq B \end{cases}$$

2.1.4 Up and Out Call/Put

To have a down and in option, the barrier is set such that $M_s < B$ and if during the life of the option, the asset price $M_s \geq B$ then the option becomes inactive and the holder is eligible to receive a rebate. The payoff of an Up and in (UI) can be expressed as:

$$UO_{call} = \begin{cases} [S_T - K]^+, & M_s < B \\ R, & M_s \geq B \end{cases}$$

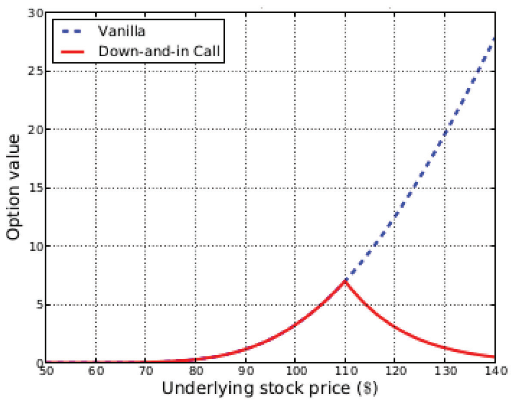
$$UO_{put} = \begin{cases} [K - S_T]^+, & M_s < B \\ 0, & M_s \geq B \end{cases}$$

2.2. Relationship between Barrier Option and vanilla Option

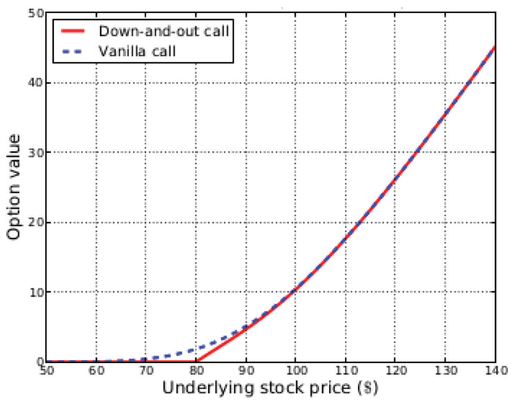
Considering the barrier option with no rebate, then we can obtain some relationship between the vanilla and barrier option. Suppose that we have a portfolio comprised of a down and in together with down and out call option. If the barrier is not hit during the life of the option, then the down and out call option will provide us the vanilla call otherwise, the down and out call expires worthless and the down and in call will provide the vanilla call. So,

$$\text{Vanilla Call} = \text{Down and In Call} + \text{Down and Out Call}$$

This can be illustrated further with the help of a diagram



(a) Down-and-in call versus vanilla call



(b) Down-and-out call versus vanilla call

Figure 1: A figure showing the relationship barrier option and vanilla option

Similarly, for the other barrier options we have the following relationships

$$\text{Vanilla Call} = \text{Up and In Call} + \text{Up and Out Call}$$

$$\text{Vanilla Put} = \text{Down and In Put} + \text{Down and Out Put}$$

$$\text{Vanilla Put} = \text{Up and In Put} + \text{Up and Out Put}$$

3. Finite Difference Approximation

This section presents Crank Nicolson finite difference method for the valuation of barrier options. We consider a security which depends on single stochastic variable S_t. Suppose that the underlying asset follows a Geometric Brownian Motion (GBM), then

$$dS_t = S_t\left(\mu dt + \sigma dW_t\right) \tag{1}$$

where dW_t is the Brownian motion, μ and σ are both constants which represent the drift and the volatility of the underlying asset. The value of the derivative security f must satisfy the Black Scholes partial differential equation given in [10].

$$\frac{\partial f}{\partial t} + rS\frac{\partial f}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 f}{\partial S^2} - rf = 0 \tag{2}$$

[11] indicates that finite difference with logarithmic transformation is the most efficient approach when stock options of large numbers are being considered. To obtain the log transform of (2), we define:

$$x = \ln S$$

Therefore,

$$\frac{\partial f}{\partial S} = \frac{1}{S}\frac{\partial f}{\partial x}$$
$$\frac{\partial^2 f}{\partial S^2} = \frac{1}{S^2}\left(\frac{\partial^2 f}{\partial x^2} - \frac{\partial f}{\partial x}\right)$$

So, (2) becomes

$$\frac{\partial f}{\partial t} + \left(r - \frac{1}{2}\sigma^2\right)\frac{\partial f}{\partial x} + \frac{1}{2}\sigma^2\frac{\partial^2 f}{\partial x^2} - rf = 0 \tag{3}$$

However, the standard practice in numerical procedures for solving partial differential equations is to simplify the equation further so as to reduce it to a dimensionless form. In doing this, we use the transformation similar to that defined by [7]

$$f(t,x) \equiv g(\tau, y)e^{-\tau r}$$
$$y \equiv \frac{x}{\sigma}$$
$$\tau = T - t$$

Thus,

$$\frac{\partial f}{\partial t} = -\frac{\partial f}{\partial \tau} = re^{-\tau r}g(\tau, y) - e^{-\tau r}\frac{\partial g}{\partial \tau}$$
$$\frac{\partial f}{\partial x} = \frac{1}{\sigma}\frac{\partial f}{\partial y} = \frac{1}{\sigma}e^{-\tau r}\frac{\partial g}{\partial y}$$
$$\frac{\partial^2 f}{\partial x^2} = \frac{1}{\sigma^2}\frac{\partial^f}{\partial y^2} = \frac{1}{\sigma^2}e^{-\tau r}\frac{\partial^2 g}{\partial y^2}$$

Hence (3) reduces to

$$re^{-\tau r}g - e^{-\tau r}\frac{\partial g}{\partial \tau} + \left(r - \frac{1}{2}\sigma^2\right)\frac{1}{\sigma}e^{-\tau r}\frac{\partial g}{\partial y} + \frac{1}{2}\sigma^2\frac{1}{\sigma^2}e^{-\tau r}\frac{\partial^2 g}{\partial y^2} - rge^{-\tau r} = 0$$

$$rg - \frac{\partial g}{\partial \tau} + \left(r - \frac{1}{2}\sigma^2\right)\frac{1}{\sigma}\frac{\partial g}{\partial y} + \frac{1}{2}\frac{\partial^2 g}{\partial y^2} - rg = 0$$

$$\frac{\partial g}{\partial \tau} - \frac{1}{\sigma}\left(r - \frac{1}{2}\sigma^2\right)\frac{\partial g}{\partial y} - \frac{1}{2}\frac{\partial^2 g}{\partial y^2} = 0$$

$$\frac{\partial g}{\partial \tau} - \gamma\frac{\partial g}{\partial y} - \frac{1}{2}\frac{\partial^2 g}{\partial y^2} = 0 \tag{4}$$

To implement the finite difference approximation, we have the nodes which can also be represented as a grid. The terminal values on the grid consist of the terminal conditions while the edges of the grid represent the boundary conditions as illustrated in [12]. Then we ensure that we have an evenly spaced grid on the (t, y)-space. To solve (4), a discrete time grid of size say (M by N) is set up to reflect the option prices over the course of time such that both S and t take on the following values at each point on the grid.

$$y = y_0, y_0 + \Delta y, y_0 + \Delta 2y, y_0 + \Delta 3y, \cdots, y_0 + (M-1)\Delta y, y_{\max}$$
$$t = t_0, t_0 + \Delta t, t_0 + \Delta 2t, t_0 + \Delta 3t, \cdots, t_0 + (N-1)\Delta t, T.$$

where T is the maturity date and both on the space. It follows by the grid notation that

$$g_i^j = g(idy, jdt)$$

$S_{\max}$ is a suitably large asset price that cannot be reached by the maturity time, T. Δy and Δt are small interval between each node in the grid, incremented by price and time respectively. The terminal condition at expiration time T for every value of S is $\max(S - K, 0)$ for a call option with strike K and $\max(K - S, 0)$ for a put option. The grid traverses backward from the terminal conditions, complying with the Partial differential equation while adhering to the boundary conditions of the grid, such as the payoff from an early exercise. The boundary conditions are defined values at the extreme ends of the nodes, where $i = 0$ and $i = N$ for every time at t. Values at the boundaries are used to calculate the values of all other lattice nodes iteratively using the partial differential equation. A visual representation of the grid is given by the following figure. As i and j increase from the top-left corner of the grid, the price S tends toward $S_{\max}$ (the maximum price possible) at the bottom-right corner of the grid:

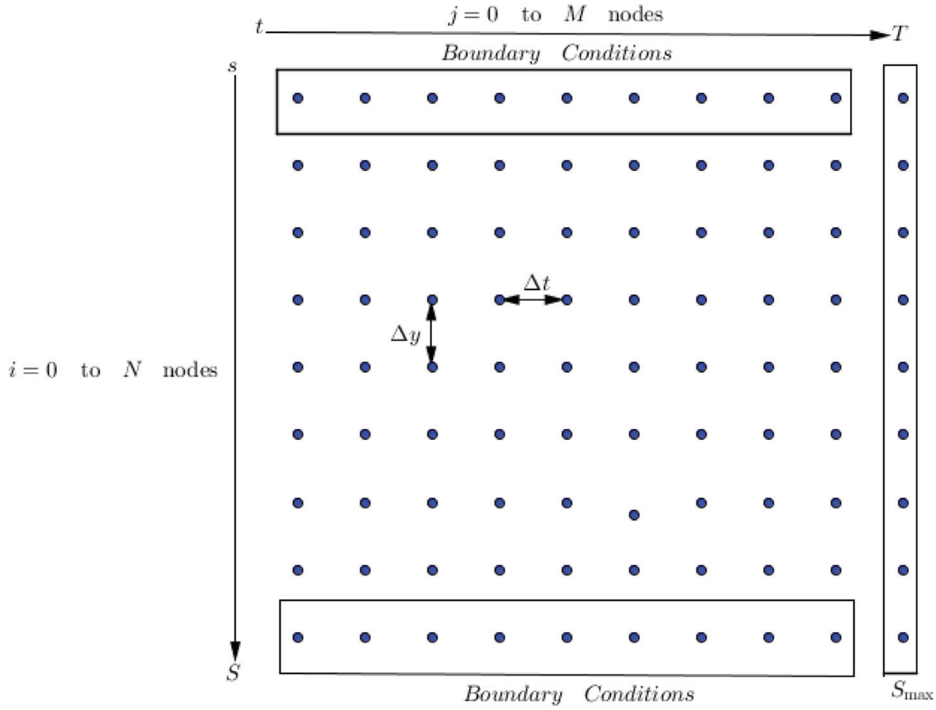
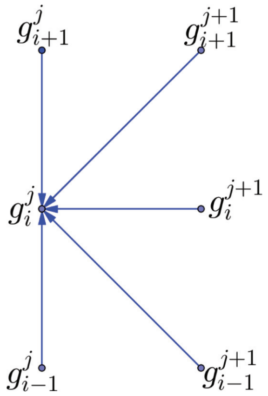


Figure 2: Visual Representation of the Grids

To obtain the finite difference approximation of (3), we use the combination of both the explicit and implicit method then taking the average of both. we therefore have

$$
\begin{aligned}
\frac{\partial g}{\partial \tau} &= \frac{g_i^{j+1} - g_i^j}{\Delta \tau} \\
\frac{\partial g}{\partial y} &= \frac{g_{i+1}^{j+1} - g_{i-1}^{j+1} + g_{i+1}^{j} - g_{i-1}^{j}}{4\Delta y} \\
\frac{\partial^2 g}{\partial y^2} &= \frac{g_{i+1}^{j+1} - 2g_i^{j+1} + g_{i-1}^{j+1} + g_{i+1}^{j} - 2g_i^{j} + g_{i-1}^{j}}{2\Delta y^2}
\end{aligned}
$$

The iterative approach of the Crank Nicolson scheme can be represented by the figure below



substituting the above into equation (3), we have

$$\begin{aligned}
&\frac{1}{\Delta\tau}\left[g_i^{j+1}-g_i^j\right]+\frac{\gamma}{4\Delta y}\left[g_{i+1}^{j+1}-g_{i-1}^{j+1}+g_{i+1}^{j}-g_{i-1}^{j}\right]\\
&\quad+\frac{1}{4\Delta y^2}\left[g_{i+1}^{j+1}-2g_i^{j+1}+g_{i-1}^{j+1}+g_{i+1}^{j}-2g_i^j+g_{i-1}^{j}\right]-\frac{r}{2}\left[g_i^{j+1}+g_i^j\right]=0\\
&g_{i+1}^{j}\left[\frac{-\gamma}{4\Delta y}-\frac{1}{4\Delta y^2}\right]+g_{i+1}^{j+1}\left[\frac{-\gamma}{4\Delta y}-\frac{1}{4\Delta y^2}\right]+g_i^{j+1}\left[\frac{1}{\Delta\tau}+\frac{1}{2\Delta y^2}\right]\\
&\quad+g_i^j\left[-\frac{1}{\Delta\tau}+\frac{1}{2\Delta y^2}\right]+g_{i-1}^{j+1}\left[\frac{\gamma}{4\Delta y}-\frac{1}{4\Delta y^2}\right]+g_{i-1}^{j}\left[\frac{\gamma}{4\Delta y}-\frac{1}{4\Delta y^2}\right]=0\\
&g_{i+1}^{j}\left[\frac{-\gamma\Delta\tau}{4\Delta y}-\frac{\Delta\tau}{4\Delta y^2}\right]+g_{i+1}^{j+1}\left[\frac{-\gamma\Delta\tau}{4\Delta y}-\frac{\Delta\tau}{4\Delta y^2}\right]+g_i^{j+1}\left[1+\frac{\Delta\tau}{2\Delta y^2}\right]\\
&\quad+g_i^j\left[-1+\frac{\Delta\tau}{2\Delta y^2}\right]+g_{i-1}^{j+1}\left[\frac{\gamma\Delta\tau}{4\Delta y}-\frac{\Delta\tau}{4\Delta y^2}\right]+g_{i-1}^{j}\left[\frac{\gamma\Delta\tau}{4\Delta y}-\frac{\Delta\tau}{4\Delta y^2}\right]=0\\
&g_{i-1}^{j+1}\left[\frac{\gamma\Delta\tau}{4\Delta y}-\frac{\Delta\tau}{4\Delta y^2}\right]+g_i^{j+1}\left[1+\frac{\Delta\tau}{2\Delta y^2}\right]+g_{i+1}^{j+1}\left[\frac{-\gamma\Delta\tau}{4\Delta y}-\frac{\Delta\tau}{4\Delta y^2}\right]\\
&\quad=g_{i-1}^{j}\left[\frac{\Delta\tau}{4\Delta y^2}-\frac{\gamma\Delta\tau}{4\Delta y}\right]+g_i^j\left[1-\frac{\Delta\tau}{2\Delta y^2}\right]g_{i+1}^{j}\left[\frac{\gamma\Delta\tau}{4\Delta y}+\frac{\Delta\tau}{4\Delta y^2}\right]
\end{aligned}$$

$$\alpha_{i-1}g_{i-1}^{j+1}+\alpha_i g_i^{j+1}+\alpha_{i+1}g_{i+1}^{j+1}=\beta_{i-1}g_{i-1}^{j}+\beta_i g_i^j+\beta_{i+1}g_{i+1}^{j} \tag{5}$$

where

$$\begin{aligned}
\alpha_{i-1}&=\frac{\gamma\Delta\tau}{4\Delta y}-\frac{\Delta\tau}{4\Delta y^2}, &\beta_{i-1}&=\frac{\Delta\tau}{4\Delta y^2}-\frac{\gamma\Delta\tau}{4\Delta y}\\
\alpha_i&=1+\frac{\Delta\tau}{2\Delta y^2}, &\beta_i&=1-\frac{\Delta\tau}{2\Delta y^2}\\
\alpha_{i+1}&=\frac{-\gamma\Delta\tau}{4\Delta y}-\frac{\Delta\tau}{4\Delta y^2}, &\beta_{i+1}&=\frac{\gamma\Delta\tau}{4\Delta y}+\frac{\Delta\tau}{4\Delta y^2}\\
&&\gamma&=\frac{1}{\sigma}\left(r-\frac{1}{2}\sigma^2\right)
\end{aligned}$$

4. Stability Analysis

The three fundamental factors that characterize a numerical scheme are consistency, stability and convergence [13, 14]. However, these factors are linked by Lax equivalence theorem [15] which states necessary and sufficient condition for convergence is stability for a well posed problem and a consistent scheme.

Necessary and Sufficient Condition for Stability

From (5), we have that the finite difference equation relating the mesh-point values along the j^{th} and $(j+1)^{th}$ time rows is given as

$$\alpha_{i-1}g_{i-1}^{j+1} + \alpha_i g_i^{j+1} + \alpha_{i+1}g_{i+1}^{j+1} = \beta_{i-1}g_{i-1}^{j} + \beta_i g_i^{j} + \beta_{i+1}g_{i+1}^{j}$$

where the coefficients are constants. Also, let the boundary values at $j > 0,\ i = 0$ and N are known, then the $(N-1)$ equations for $i = 1, 2, \cdots, N-1$ can be represented in a matrix form as

$$\begin{bmatrix} \alpha_1 & \alpha_2 & & & \\ \alpha_1 & \alpha_2 & \alpha_3 & & \\ & \ddots & \ddots & \ddots & \\ & & \alpha_{N-3} & \alpha_{N-2} & \alpha_{N-1} \\ & & & \alpha_{N-2} & \alpha_{N-1} \end{bmatrix} \begin{bmatrix} g_1^{j+1} \\ g_2^{j+1} \\ \vdots \\ g_{N-2}^{j+1} \\ g_{N-1}^{j+1} \end{bmatrix}$$
$$= \begin{bmatrix} \beta_1 & \beta_2 & & & \\ \beta_1 & \beta_2 & \beta_3 & & \\ & \ddots & \ddots & \ddots & \\ & & \beta_{N-3} & \beta_{N-2} & \beta_{N-1} \\ & & & \beta_{N-2} & \beta_{N-1} \end{bmatrix} \begin{bmatrix} g_1^{j} \\ g_2^{j} \\ \vdots \\ g_{N-2}^{j} \\ g_{N-1}^{j} \end{bmatrix} + \begin{bmatrix} \beta_0 g_0^j - \alpha_0 g_0^{j+1} \\ 0 \\ \vdots \\ 0 \\ \beta_N g_N^j - \alpha_N g_N^{j+1} \end{bmatrix}$$

which is of the form

$$\begin{aligned} Ag_{j+1} &= Bg_j + d_j \\ g_{j+1} &= A^{-1}Bg_j + A^{-1}d_j \end{aligned} \tag{6}$$

where A and B are matrices of order $N-1$. g_{j+1} and g_j denotes the column vectors.

Hence (6) can be expressed as

$$g_{j+1} = Ug_j + V_j$$

where $U = A^{-1}B$ and $V_j = A^{-1}d_j$.

Thus

$$
\begin{aligned}
g_j &= Ug_{j-1} + V_{j-1} \\
&= U^2 g_{j-2} + UV_{j-2} + V_{j-1} \\
&\vdots \\
&= U^j g_0 + U^{j-1}V_0 + U^{j-2}V_1 + \cdots + V_{j-1}
\end{aligned} \tag{7}
$$

g_0 is the vector of initial values and $V_0, V_1, \cdots, V_{j-1}$ are vectors of known boundary values. According to [15], we perturbed the vector of the initial value g_0 to $\widehat{g}_0$. So at the j^{th} row, the exact solution becomes

$$\widehat{g}_0 = U^j \widehat{g}_0 + U^{j-1}V_0 + U^{j-2}V_1 + \cdots + V_{j-1} \tag{8}$$

and we define the perturbation e as

$$e = \widehat{g} - g$$

using (7) and (8), we have

$$
\begin{aligned}
e_j &= \widehat{g}_j - g_j \\
&= U^j(\widehat{g}_0 - g_0) \\
&= U^j e_0
\end{aligned}
$$

Hence for compatible matrix and vector norms,

$$\|e_j\| \leq \|U^j\| \|e_0\|$$

Thus according to Lax and Richtmyer [16], a difference scheme is said to be stable if there exists a positive number L independent of j, $\Delta\tau$ and Δy, such that

$$\|U\| \leq L$$

Since

$$\|e_j\| \leq L\|e_0\|$$

and it follows that the Lax-Richmyer definition of stability is satisfied by

$$\|U\| \leq 1 \tag{9}$$

which is the necessary and sufficient condition for the finite difference equations to be stable. The matrix method of analysis shows that the equations are stable if the largest of the moduli of the engenvalues of matrix U (spectral radius) satisfies

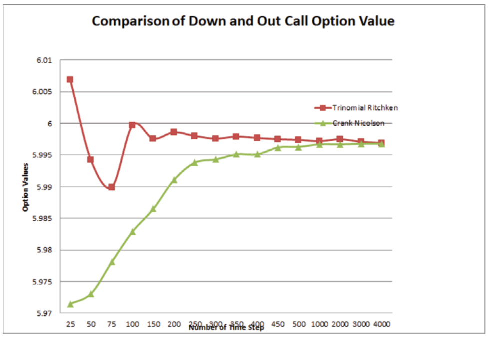
$$\rho(U) \leq 1.$$

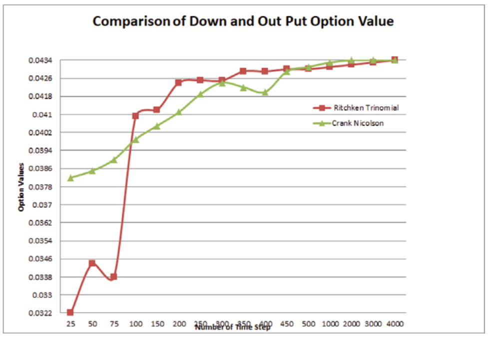
Since the spectral radius denoted by $\rho(U)$ satisfies $\rho(U) \leq U$, then it follows that $\rho(U) \leq 1$.

Therefore Lax equivalence theorem, the Crank-Nicolson finite difference methods are unconditionally stable, convergent and consistent. For a small change in the initial conditions, the scheme calculate small change in the option value and increases in the number of time steps and price partitions decreases the calculation error.

Table 1: Down and Out Call Option Prices

	Ritchken Trinomial		Crank Nicolson FDM	
No of time partition	Call Option	Put Option	Call Option	Put Option
25	6.0069	0.0322	5.9715	0.0382
50	5.9942	0.0344	5.9731	0.0385
75	5.9899	0.0338	5.9781	0.0390
100	5.9997	0.0409	5.9829	0.0.399
150	5.9976	0.0412	5.9865	0.0405
200	5.9986	0.0424	5.9911	0.0411
250	5.9980	0.0425	5.9938	0.0419
300	5.9976	0.0425	5.9943	0.0424
350	5.9979	0.0429	5.9951	0.0422
400	5.9977	0.0429	5.9951	0.0424
450	5.9975	0.0429	5.9962	0.0429
500	5.9974	0.0430	5.9963	0.0431
1000	5.9972	0.0430	5.9967	0.0433
2000	5.9970	0.0431	5.9967	0.0434
3000	5.9969	0.0432	5.9968	0.0434
4000	5.9968	0.0433	5.9968	0.0434
Exact Value	**5.9968**	**0.0434**	**5.9968**	**0.0434**



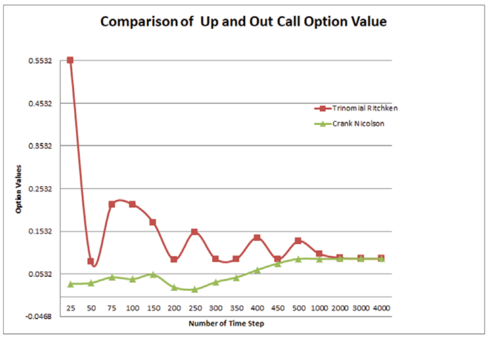


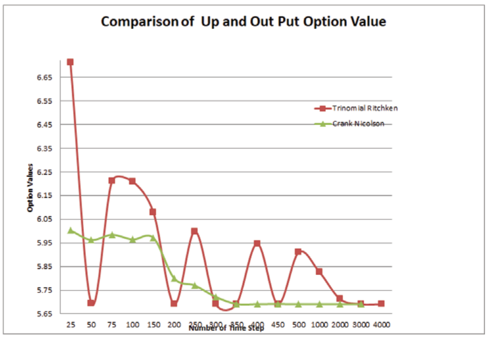
5. Numerical Example

In this section, we shall consider the knock out option. At any time during the life of the option, should the underlying asset price fall below the barrier price, the option is considered worthless. The same parameter [6] is used. We consider an underlying asset whose current price is 95, the strike price is 100, the volatility of the underlying asset is 25% per annum with 1 year remaining to the maturity date, the risk-free interest rate is 10% per annum while the barrier is set at 90. A numerical result is presented in Table 1 which helps to illustrate the option value obtained when considering different step size. The table reports the prices of the Down and Out call option using Ritchken trinomial [6]

Table 2: Up and Out Call Option Prices

No of time partition	Ritchken Trinomial		Crank Nicolson FDM	
	Call Option	Put Option	Call Option	Put Option
25	0.5532	6.7117	0.0301	6.0031
50	0.0826	5.6957	0.0322	5.9623
75	0.2156	6.2105	0.0453	5.9843
100	0.2163	6.2087	0.0409	5.9645
150	0.1735	6.0789	0.0513	5.9711
200	0.0867	5.6913	0.0218	5.8001
250	0.1507	5.9988	0.0172	5,7699
300	0.0879	5.6916	0.0342	5.7219
350	0.0880	5.6915	0.0451	5.6918
400	0.1382	5.9468	0.0624	5.6912
450	0.0879	5.6909	0.0781	5.6915
500	0.1302	5.9103	0.0886	5.6910
1000	0.1002	5.8263	0.0887	5.6908
2000	0.0908	5.7123	0.0888	5.6908
3000	0.0891	5.6909	0.0888	5.6907
4000	0.0891	5.6909	0.0889	5.6907
Exact Value	**0.0889**	**5.6907**	**0.0889**	**5.6907**





and when using the Crank Nicolson method which involved taking the average of both the explicit and implicit finite difference. From the table, it is observed that we obtain the closed form solution faster compared to the other procedure given in the table.

Similarly, for the case of the Up and Out option, the same parameters are used with the exception of the position of the barrier which is chosen to be 110 so that the boundary conditions could be satisfied. The numerical result obtained is provided in the table 2.

From Table 2, it is observed that the result obtained using the trinomial model does not converge to the accurate value of the option and also, the method involves alot of computation and larger time step before the option value could move close to the accurate value whereas in the Crank Nicolson approach used in this work with fewer time step and less computation,the result obtained converges faster to the the accurate value.

6. Conclusion

In this study, we have extend the approach that are used in the valuation of the barrier options. It has been recognised that the trinomial tree method which is also the improvement of the binomial method is not so accurate for the valuation of the barrier option due to the fact that the price of such option is very sensitive to the location of the barrier. So, this study addressed the problem and as such make use of the finite difference method. The three methods presented are : explicit, implicit and Crank Nicolson. The Explicit method is very easy to implement but the encounter the stability problem. Overcoming stability issues leads to the introduction of the implicit method and even more precise Crank Nicolson method. However from our numerical results, we observed that the Crank Nicolson finite difference method is a more suitable approach for the valuation of the barrier options as it gives a more accurate result and the reduces the number of time steps before the convergence to the exact option value.

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