



MACHAKOS UNIVERSITY

University Examinations for 2019/2020 Academic Year

SCHOOL OF AGRICULTURAL SCIENCES

DEPARTMENT OF AGRIBUSINESS MANAGEMENT AND TRADE

FIRST YEAR SECOND SEMESTER EXAMINATION FOR

MASTER OF SCIENCE IN AGRIBUSINESS MANAGEMENT

AGB 802: QUANTITATIVE METHODS IN AGRIBUSINESS

DATE: 26/2/2021

TIME: 9.00-12.00 PM

INSTRUCTIONS:

Answer **QUESTION ONE** and **TWO other** questions. Clearly show all your workings. Statistical tables have been provided.

QUESTION ONE (20 MARKS)

- a) A project activity has most likely, pessimistic and optimistic time estimates of 17, 25 and 15 days, respectively. Compute the activity's
- Mean duration (1 mark)
 - Standard deviation of the duration (1 mark)
- b) The average number of customers arriving at a supermarket in a 60-minute period of time is 152. What is the probability of exactly 135 arrivals in 60 minutes? (2 marks)
- c) A sample of 34 packets of rice returned a mean of 23.5 Kg and standard deviation of 0.45Kg. Does the sample mean support claims that the population mean is 25.0Kg, at the 98 percent confidence interval? (3 marks)
- d) A company uses 2,600 packets of a certain pesticide per year, which costs KSh 625 each. The ordering and handling costs are KSh 3750 per order and carrying costs are 7.5% of purchase price per annum. Compute the economic order quantity (3 marks)
- e) Using the data below:
- find the correlation coefficient (3 marks)
 - Interpret the results (1 mark)

| | | | | | | |
|----------------------------|-----|-----|-----|-----|-----|-----|
| Labour costs (Ksh Million) | 21 | 23 | 24 | 25 | 26 | 27 |
| Profit (KSh Million) | 182 | 171 | 156 | 144 | 123 | 110 |

- f) Below is data on number of employees from surveyed companies. Find the:
- Mean number of employees (3 marks)
 - Median number of employees (3 marks)

| | | | | | |
|------------------|---------|-----------|-------------|-------------|-------------|
| No. of employees | 100-500 | 500-9,000 | 9,000-1,300 | 1,300-1,700 | 1,700-2,100 |
| Frequency (f) | 19 | 37 | 58 | 31 | 12 |

QUESTION TWO (20 MARKS)

- a) A cereal trader intends to distribute rice from his four warehouses to four towns at the lowest possible cost. Quantities available at the warehouses are 100, 80, 150 and 80 tons respectively, while wheat demand is 120, 90, 60, and 140 tons in the four towns, respectively. It will cost the trader KSh 5500, 4000, 1000 and 2500 to ship a ton of wheat from the first warehouse to towns 1,2,3 and 4; KSh 3000, 5000, 4500 and 2,000 to ship a ton of wheat from the second warehouse to towns 1,2,3 and 4; KSh 4000, 4700, 1500 and 2,000 to ship a ton of wheat from the third warehouse to towns 1,2,3 and 4, respectively, and KSh 3500, 1250, 6500 and 1,800 to ship a ton of wheat from the fourth warehouse to towns 1,2,3 and 4, respectively.
- Advise the trader on the best transport routes using:
- The Intuitive Lowest-Cost Method (4 marks)
 - The north West Corner Rule (4 marks)
- b) Suppose a firm is producing two types of products whose profits per Kg are KSh 15 and Ksh 20 respectively. The two products require three types of inputs: land, labor and capital, as shown in the table below.
- Formulate the problem as a linear programming model (3 marks)
 - Using the simplex method, advise the firm (6 marks)
 - Comment on resource use (3 marks)

| | | | |
|--------------------|-------------|-------------|-----------------|
| Profit/Kg | P1 (KSh 30) | P2 (KSh 40) | Total available |
| Land (acres) | 3 | 2 | 1200 |
| Labor (man days) | 30 | 50 | 1600 |
| Capital (KSh '000) | 5 | 6 | 2200 |

QUESTION THREE (20 MARKS)

- a) An employee claims that monthly total earnings per worker from two of your farm’s branches are not the same. The human resource manager samples 23 workers from one branch and 17 workers from the second branch, and obtains mean earnings of KSh 15630 and KSh 18310 respectively. The sample standard deviation of branch1 earnings is KSh 1230 while that of branch2 earnings is KSh 1290. Will the human resource manager find the employee’s claim statistically justified? (8 marks)
- b) The following table gives data at normal time and cost-crashed time and project cost.
- Draw the project network using the activity-on-arrow approach (4 marks)
 - Determine a crashing scheme for the project so that the total project time is reduced by 12 weeks (4 marks)
 - What is the overall project duration after crashing? (2 marks)
 - What is the optimal project cost after crashing? (2 marks)

| Activity | Predecessor Activity | Time (wks) | | Cost (KSh) | |
|----------|----------------------|------------|-------|------------|-------|
| | | Normal | Crash | Normal | Crash |
| A | | 4 | 3 | 8000 | 9000 |
| B | A | 5 | 3 | 16000 | 20000 |
| C | A | 4 | 3 | 12000 | 13000 |
| D | B | 6 | 5 | 34000 | 35000 |
| E | C,D | 6 | 4 | 42000 | 44000 |
| F | B,E | 5 | 4 | 16000 | 16500 |

QUESTION FOUR (20 MARKS)

- a) The table below shows the number of agribusiness firms accessing credit. Test whether credit access differs among men, women and the youth (8 marks)

| Purchase Decision | Women | Men | Youth |
|--|-------|-----|-------|
| Number of traders accessing credit | 18 | 32 | 4 |
| Number of traders not accessing credit | 54 | 50 | 34 |

- b) The data below was extracted from records of Kuku Ltd.
- Develop a linear regression equation for estimating egg production (10 marks)
 - Predict egg production if the firm spends KSh 40,000 on feeds (2 marks)

| | | | | | |
|----------------------------|---|----|----|----|----|
| Feed costs (Ksh ‘000) | 6 | 11 | 17 | 27 | 32 |
| Egg production (x10 trays) | 3 | 5 | 8 | 12 | 15 |

QUESTION FIVE (20 MARKS)

- a) Below is data showing milk production by dairy farmers in three counties. Does the milk production per farmer differ across the counties? (12 marks)

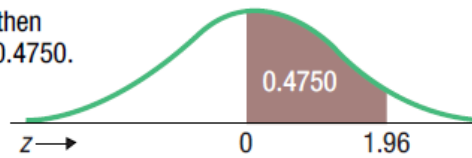
| Milk production per farmer (Litres) | | | |
|-------------------------------------|----------|----------|----------|
| County A | County B | County C | County D |
| 5 | 17 | 3 | 17 |
| 7 | 15 | 21 | 16 |
| 10 | 36 | 13 | 8 |
| 6 | 25 | 9 | 5 |

- b) A farmer must decide on the enterprise to invest in, to maximize returns in her 30 acres farm. The farmer can produce maize, kales, sorghum or tomatoes. Crop output is dependent on weather. When rains are average, the farmer can produce 0.7, 1.2, 1.5 and 0.9 tons per acre of maize, kales, sorghum and tomatoes, respectively. Yields under high rainfall are 1.9, 1.6, 1.7 and 2.8 tons per acre, respectively, while under poor rains, the yields are 0.3, 0.5, 1.2 and 0.5 tons per acre, respectively. The table below shows rainfall probabilities and associated grain prices. Advise the farmer based on:
- The pessimistic approach (2 marks)
 - The expected value approach (8 marks)

| Enterprise | High Rainfall (probability = 0.2) | Low Rainfall (probability=0.3) | Medium Rainfall Probability = 0.5) |
|------------|--------------------------------------|-----------------------------------|---------------------------------------|
| | Price (Ksh/kg) | Price (Ksh/kg) | Price (KSh/kg) |
| Maize | 21 | 35 | 25 |
| Kales | 15 | 45 | 30 |
| Sorghum | 35 | 60 | 45 |
| Tomatoes | 50 | 120 | 75 |

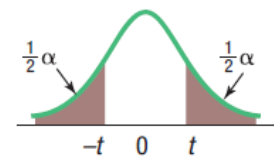
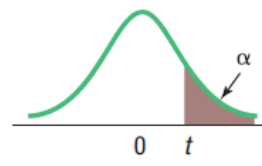
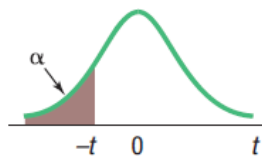
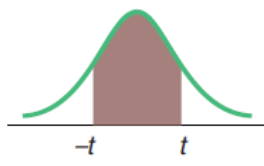
B.1 Areas under the Normal Curve

Example:
If $z = 1.96$, then
 $P(0 \text{ to } z) = 0.4750$.



| z | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
|-----|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 0.0 | 0.0000 | 0.0040 | 0.0080 | 0.0120 | 0.0160 | 0.0199 | 0.0239 | 0.0279 | 0.0319 | 0.0359 |
| 0.1 | 0.0398 | 0.0438 | 0.0478 | 0.0517 | 0.0557 | 0.0596 | 0.0636 | 0.0675 | 0.0714 | 0.0753 |
| 0.2 | 0.0793 | 0.0832 | 0.0871 | 0.0910 | 0.0948 | 0.0987 | 0.1026 | 0.1064 | 0.1103 | 0.1141 |
| 0.3 | 0.1179 | 0.1217 | 0.1255 | 0.1293 | 0.1331 | 0.1368 | 0.1406 | 0.1443 | 0.1480 | 0.1517 |
| 0.4 | 0.1554 | 0.1591 | 0.1628 | 0.1664 | 0.1700 | 0.1736 | 0.1772 | 0.1808 | 0.1844 | 0.1879 |
| 0.5 | 0.1915 | 0.1950 | 0.1985 | 0.2019 | 0.2054 | 0.2088 | 0.2123 | 0.2157 | 0.2190 | 0.2224 |
| 0.6 | 0.2257 | 0.2291 | 0.2324 | 0.2357 | 0.2389 | 0.2422 | 0.2454 | 0.2486 | 0.2517 | 0.2549 |
| 0.7 | 0.2580 | 0.2611 | 0.2642 | 0.2673 | 0.2704 | 0.2734 | 0.2764 | 0.2794 | 0.2823 | 0.2852 |
| 0.8 | 0.2881 | 0.2910 | 0.2939 | 0.2967 | 0.2995 | 0.3023 | 0.3051 | 0.3078 | 0.3106 | 0.3133 |
| 0.9 | 0.3159 | 0.3186 | 0.3212 | 0.3238 | 0.3264 | 0.3289 | 0.3315 | 0.3340 | 0.3365 | 0.3389 |
| 1.0 | 0.3413 | 0.3438 | 0.3461 | 0.3485 | 0.3508 | 0.3531 | 0.3554 | 0.3577 | 0.3599 | 0.3621 |
| 1.1 | 0.3643 | 0.3665 | 0.3686 | 0.3708 | 0.3729 | 0.3749 | 0.3770 | 0.3790 | 0.3810 | 0.3830 |
| 1.2 | 0.3849 | 0.3869 | 0.3888 | 0.3907 | 0.3925 | 0.3944 | 0.3962 | 0.3980 | 0.3997 | 0.4015 |
| 1.3 | 0.4032 | 0.4049 | 0.4066 | 0.4082 | 0.4099 | 0.4115 | 0.4131 | 0.4147 | 0.4162 | 0.4177 |
| 1.4 | 0.4192 | 0.4207 | 0.4222 | 0.4236 | 0.4251 | 0.4265 | 0.4279 | 0.4292 | 0.4306 | 0.4319 |
| 1.5 | 0.4332 | 0.4345 | 0.4357 | 0.4370 | 0.4382 | 0.4394 | 0.4406 | 0.4418 | 0.4429 | 0.4441 |
| 1.6 | 0.4452 | 0.4463 | 0.4474 | 0.4484 | 0.4495 | 0.4505 | 0.4515 | 0.4525 | 0.4535 | 0.4545 |
| 1.7 | 0.4554 | 0.4564 | 0.4573 | 0.4582 | 0.4591 | 0.4599 | 0.4608 | 0.4616 | 0.4625 | 0.4633 |
| 1.8 | 0.4641 | 0.4649 | 0.4656 | 0.4664 | 0.4671 | 0.4678 | 0.4686 | 0.4693 | 0.4699 | 0.4706 |
| 1.9 | 0.4713 | 0.4719 | 0.4726 | 0.4732 | 0.4738 | 0.4744 | 0.4750 | 0.4756 | 0.4761 | 0.4767 |
| 2.0 | 0.4772 | 0.4778 | 0.4783 | 0.4788 | 0.4793 | 0.4798 | 0.4803 | 0.4808 | 0.4812 | 0.4817 |
| 2.1 | 0.4821 | 0.4826 | 0.4830 | 0.4834 | 0.4838 | 0.4842 | 0.4846 | 0.4850 | 0.4854 | 0.4857 |
| 2.2 | 0.4861 | 0.4864 | 0.4868 | 0.4871 | 0.4875 | 0.4878 | 0.4881 | 0.4884 | 0.4887 | 0.4890 |
| 2.3 | 0.4893 | 0.4896 | 0.4898 | 0.4901 | 0.4904 | 0.4906 | 0.4909 | 0.4911 | 0.4913 | 0.4916 |
| 2.4 | 0.4918 | 0.4920 | 0.4922 | 0.4925 | 0.4927 | 0.4929 | 0.4931 | 0.4932 | 0.4934 | 0.4936 |
| 2.5 | 0.4938 | 0.4940 | 0.4941 | 0.4943 | 0.4945 | 0.4946 | 0.4948 | 0.4949 | 0.4951 | 0.4952 |
| 2.6 | 0.4953 | 0.4955 | 0.4956 | 0.4957 | 0.4959 | 0.4960 | 0.4961 | 0.4962 | 0.4963 | 0.4964 |
| 2.7 | 0.4965 | 0.4966 | 0.4967 | 0.4968 | 0.4969 | 0.4970 | 0.4971 | 0.4972 | 0.4973 | 0.4974 |
| 2.8 | 0.4974 | 0.4975 | 0.4976 | 0.4977 | 0.4977 | 0.4978 | 0.4979 | 0.4979 | 0.4980 | 0.4981 |
| 2.9 | 0.4981 | 0.4982 | 0.4982 | 0.4983 | 0.4984 | 0.4984 | 0.4985 | 0.4985 | 0.4986 | 0.4986 |
| 3.0 | 0.4987 | 0.4987 | 0.4987 | 0.4988 | 0.4988 | 0.4989 | 0.4989 | 0.4989 | 0.4990 | 0.4990 |

B.2 Student's *t* Distribution

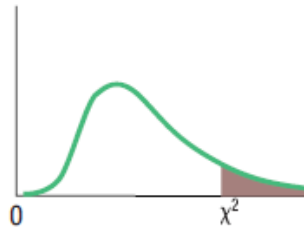


| Confidence Intervals, <i>c</i> | | | | | | |
|--------------------------------|---|-------|--------|--------|--------|---------|
| <i>df</i> | 80% | 90% | 95% | 98% | 99% | 99.9% |
| | Level of Significance for One-Tailed Test, α | | | | | |
| | 0.10 | 0.05 | 0.025 | 0.01 | 0.005 | 0.0005 |
| | Level of Significance for Two-Tailed Test, α | | | | | |
| | 0.20 | 0.10 | 0.05 | 0.02 | 0.01 | 0.001 |
| 1 | 3.078 | 6.314 | 12.706 | 31.821 | 63.657 | 636.619 |
| 2 | 1.886 | 2.920 | 4.303 | 6.965 | 9.925 | 31.599 |
| 3 | 1.638 | 2.353 | 3.182 | 4.541 | 5.841 | 12.924 |
| 4 | 1.533 | 2.132 | 2.776 | 3.747 | 4.604 | 8.610 |
| 5 | 1.476 | 2.015 | 2.571 | 3.365 | 4.032 | 6.869 |
| 6 | 1.440 | 1.943 | 2.447 | 3.143 | 3.707 | 5.959 |
| 7 | 1.415 | 1.895 | 2.365 | 2.998 | 3.499 | 5.408 |
| 8 | 1.397 | 1.860 | 2.306 | 2.896 | 3.355 | 5.041 |
| 9 | 1.383 | 1.833 | 2.262 | 2.821 | 3.250 | 4.781 |
| 10 | 1.372 | 1.812 | 2.228 | 2.764 | 3.169 | 4.587 |
| 11 | 1.363 | 1.796 | 2.201 | 2.718 | 3.106 | 4.437 |
| 12 | 1.356 | 1.782 | 2.179 | 2.681 | 3.055 | 4.318 |
| 13 | 1.350 | 1.771 | 2.160 | 2.650 | 3.012 | 4.221 |
| 14 | 1.345 | 1.761 | 2.145 | 2.624 | 2.977 | 4.140 |
| 15 | 1.341 | 1.753 | 2.131 | 2.602 | 2.947 | 4.073 |
| 16 | 1.337 | 1.746 | 2.120 | 2.583 | 2.921 | 4.015 |
| 17 | 1.333 | 1.740 | 2.110 | 2.567 | 2.898 | 3.965 |
| 18 | 1.330 | 1.734 | 2.101 | 2.552 | 2.878 | 3.922 |
| 19 | 1.328 | 1.729 | 2.093 | 2.539 | 2.861 | 3.883 |
| 20 | 1.325 | 1.725 | 2.086 | 2.528 | 2.845 | 3.850 |
| 21 | 1.323 | 1.721 | 2.080 | 2.518 | 2.831 | 3.819 |
| 22 | 1.321 | 1.717 | 2.074 | 2.508 | 2.819 | 3.792 |
| 23 | 1.319 | 1.714 | 2.069 | 2.500 | 2.807 | 3.768 |
| 24 | 1.318 | 1.711 | 2.064 | 2.492 | 2.797 | 3.745 |
| 25 | 1.316 | 1.708 | 2.060 | 2.485 | 2.787 | 3.725 |
| 26 | 1.315 | 1.706 | 2.056 | 2.479 | 2.779 | 3.707 |
| 27 | 1.314 | 1.703 | 2.052 | 2.473 | 2.771 | 3.690 |
| 28 | 1.313 | 1.701 | 2.048 | 2.467 | 2.763 | 3.674 |
| 29 | 1.311 | 1.699 | 2.045 | 2.462 | 2.756 | 3.659 |
| 30 | 1.310 | 1.697 | 2.042 | 2.457 | 2.750 | 3.646 |
| 31 | 1.309 | 1.696 | 2.040 | 2.453 | 2.744 | 3.633 |
| 32 | 1.309 | 1.694 | 2.037 | 2.449 | 2.738 | 3.622 |
| 33 | 1.308 | 1.692 | 2.035 | 2.445 | 2.733 | 3.611 |
| 34 | 1.307 | 1.691 | 2.032 | 2.441 | 2.728 | 3.601 |
| 35 | 1.306 | 1.690 | 2.030 | 2.438 | 2.724 | 3.591 |

| Confidence Intervals, <i>c</i> | | | | | | |
|--------------------------------|---|-------|-------|-------|-------|--------|
| <i>df</i> | 80% | 90% | 95% | 98% | 99% | 99.9% |
| | Level of Significance for One-Tailed Test, α | | | | | |
| | 0.10 | 0.05 | 0.025 | 0.01 | 0.005 | 0.0005 |
| | Level of Significance for Two-Tailed Test, α | | | | | |
| | 0.20 | 0.10 | 0.05 | 0.02 | 0.01 | 0.001 |
| 36 | 1.306 | 1.688 | 2.028 | 2.434 | 2.719 | 3.582 |
| 37 | 1.305 | 1.687 | 2.026 | 2.431 | 2.715 | 3.574 |
| 38 | 1.304 | 1.686 | 2.024 | 2.429 | 2.712 | 3.566 |
| 39 | 1.304 | 1.685 | 2.023 | 2.426 | 2.708 | 3.558 |
| 40 | 1.303 | 1.684 | 2.021 | 2.423 | 2.704 | 3.551 |
| 41 | 1.303 | 1.683 | 2.020 | 2.421 | 2.701 | 3.544 |
| 42 | 1.302 | 1.682 | 2.018 | 2.418 | 2.698 | 3.538 |
| 43 | 1.302 | 1.681 | 2.017 | 2.416 | 2.695 | 3.532 |
| 44 | 1.301 | 1.680 | 2.015 | 2.414 | 2.692 | 3.526 |
| 45 | 1.301 | 1.679 | 2.014 | 2.412 | 2.690 | 3.520 |
| 46 | 1.300 | 1.679 | 2.013 | 2.410 | 2.687 | 3.515 |
| 47 | 1.300 | 1.678 | 2.012 | 2.408 | 2.685 | 3.510 |
| 48 | 1.299 | 1.677 | 2.011 | 2.407 | 2.682 | 3.505 |
| 49 | 1.299 | 1.677 | 2.010 | 2.405 | 2.680 | 3.500 |
| 50 | 1.299 | 1.676 | 2.009 | 2.403 | 2.678 | 3.496 |
| 51 | 1.298 | 1.675 | 2.008 | 2.402 | 2.676 | 3.492 |
| 52 | 1.298 | 1.675 | 2.007 | 2.400 | 2.674 | 3.488 |
| 53 | 1.298 | 1.674 | 2.006 | 2.399 | 2.672 | 3.484 |
| 54 | 1.297 | 1.674 | 2.005 | 2.397 | 2.670 | 3.480 |
| 55 | 1.297 | 1.673 | 2.004 | 2.396 | 2.668 | 3.476 |
| 56 | 1.297 | 1.673 | 2.003 | 2.395 | 2.667 | 3.473 |
| 57 | 1.297 | 1.672 | 2.002 | 2.394 | 2.665 | 3.470 |
| 58 | 1.296 | 1.672 | 2.002 | 2.392 | 2.663 | 3.466 |
| 59 | 1.296 | 1.671 | 2.001 | 2.391 | 2.662 | 3.463 |
| 60 | 1.296 | 1.671 | 2.000 | 2.390 | 2.660 | 3.460 |
| 61 | 1.296 | 1.670 | 2.000 | 2.389 | 2.659 | 3.457 |
| 62 | 1.295 | 1.670 | 1.999 | 2.388 | 2.657 | 3.454 |
| 63 | 1.295 | 1.669 | 1.998 | 2.387 | 2.656 | 3.452 |
| 64 | 1.295 | 1.669 | 1.998 | 2.386 | 2.655 | 3.449 |
| 65 | 1.295 | 1.669 | 1.997 | 2.385 | 2.654 | 3.447 |
| 66 | 1.295 | 1.668 | 1.997 | 2.384 | 2.652 | 3.444 |
| 67 | 1.294 | 1.668 | 1.996 | 2.383 | 2.651 | 3.442 |
| 68 | 1.294 | 1.668 | 1.995 | 2.382 | 2.650 | 3.439 |
| 69 | 1.294 | 1.667 | 1.995 | 2.382 | 2.649 | 3.437 |
| 70 | 1.294 | 1.667 | 1.994 | 2.381 | 2.648 | 3.435 |

B.3 Critical Values of Chi-Square

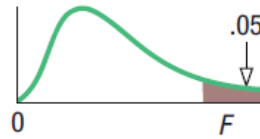
This table contains the values of χ^2 that correspond to a specific right-tail area and specific number of degrees of freedom.



Example: With 17 df and a .02 area in the upper tail, $\chi^2 = 30.995$

| Degrees of Freedom, df | Right-Tail Area | | | |
|--------------------------|-----------------|--------|--------|--------|
| | 0.10 | 0.05 | 0.02 | 0.01 |
| 1 | 2.706 | 3.841 | 5.412 | 6.635 |
| 2 | 4.605 | 5.991 | 7.824 | 9.210 |
| 3 | 6.251 | 7.815 | 9.837 | 11.345 |
| 4 | 7.779 | 9.488 | 11.668 | 13.277 |
| 5 | 9.236 | 11.070 | 13.388 | 15.086 |
| 6 | 10.645 | 12.592 | 15.033 | 16.812 |
| 7 | 12.017 | 14.067 | 16.622 | 18.475 |
| 8 | 13.362 | 15.507 | 18.168 | 20.090 |
| 9 | 14.684 | 16.919 | 19.679 | 21.666 |
| 10 | 15.987 | 18.307 | 21.161 | 23.209 |
| 11 | 17.275 | 19.675 | 22.618 | 24.725 |
| 12 | 18.549 | 21.026 | 24.054 | 26.217 |
| 13 | 19.812 | 22.362 | 25.472 | 27.688 |
| 14 | 21.064 | 23.685 | 26.873 | 29.141 |
| 15 | 22.307 | 24.996 | 28.259 | 30.578 |
| 16 | 23.542 | 26.296 | 29.633 | 32.000 |
| 17 | 24.769 | 27.587 | 30.995 | 33.409 |
| 18 | 25.989 | 28.869 | 32.346 | 34.805 |
| 19 | 27.204 | 30.144 | 33.687 | 36.191 |
| 20 | 28.412 | 31.410 | 35.020 | 37.566 |
| 21 | 29.615 | 32.671 | 36.343 | 38.932 |
| 22 | 30.813 | 33.924 | 37.659 | 40.289 |
| 23 | 32.007 | 35.172 | 38.968 | 41.638 |
| 24 | 33.196 | 36.415 | 40.270 | 42.980 |
| 25 | 34.382 | 37.652 | 41.566 | 44.314 |
| 26 | 35.563 | 38.885 | 42.856 | 45.642 |
| 27 | 36.741 | 40.113 | 44.140 | 46.963 |
| 28 | 37.916 | 41.337 | 45.419 | 48.278 |
| 29 | 39.087 | 42.557 | 46.693 | 49.588 |
| 30 | 40.256 | 43.773 | 47.962 | 50.892 |

B.4 Critical Values of the *F* Distribution at a 5 Percent Level of Significance



| | Degrees of Freedom for the Numerator | | | | | | | | | | | | | | | |
|-----|--------------------------------------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 12 | 15 | 20 | 24 | 30 | 40 |
| 1 | 161 | 200 | 216 | 225 | 230 | 234 | 237 | 239 | 241 | 242 | 244 | 246 | 248 | 249 | 250 | 251 |
| 2 | 18.5 | 19.0 | 19.2 | 19.2 | 19.3 | 19.3 | 19.4 | 19.4 | 19.4 | 19.4 | 19.4 | 19.4 | 19.4 | 19.5 | 19.5 | 19.5 |
| 3 | 10.1 | 9.55 | 9.28 | 9.12 | 9.01 | 8.94 | 8.89 | 8.85 | 8.81 | 8.79 | 8.74 | 8.70 | 8.66 | 8.64 | 8.62 | 8.59 |
| 4 | 7.71 | 6.94 | 6.59 | 6.39 | 6.26 | 6.16 | 6.09 | 6.04 | 6.00 | 5.96 | 5.91 | 5.86 | 5.80 | 5.77 | 5.75 | 5.72 |
| 5 | 6.61 | 5.79 | 5.41 | 5.19 | 5.05 | 4.95 | 4.88 | 4.82 | 4.77 | 4.74 | 4.68 | 4.62 | 4.56 | 4.53 | 4.50 | 4.46 |
| 6 | 5.99 | 5.14 | 4.76 | 4.53 | 4.39 | 4.28 | 4.21 | 4.15 | 4.10 | 4.06 | 4.00 | 3.94 | 3.87 | 3.84 | 3.81 | 3.77 |
| 7 | 5.59 | 4.74 | 4.35 | 4.12 | 3.97 | 3.87 | 3.79 | 3.73 | 3.68 | 3.64 | 3.57 | 3.51 | 3.44 | 3.41 | 3.38 | 3.34 |
| 8 | 5.32 | 4.46 | 4.07 | 3.84 | 3.69 | 3.58 | 3.50 | 3.44 | 3.39 | 3.35 | 3.28 | 3.22 | 3.15 | 3.12 | 3.08 | 3.04 |
| 9 | 5.12 | 4.26 | 3.86 | 3.63 | 3.48 | 3.37 | 3.29 | 3.23 | 3.18 | 3.14 | 3.07 | 3.01 | 2.94 | 2.90 | 2.86 | 2.83 |
| 10 | 4.96 | 4.10 | 3.71 | 3.48 | 3.33 | 3.22 | 3.14 | 3.07 | 3.02 | 2.98 | 2.91 | 2.85 | 2.77 | 2.74 | 2.70 | 2.66 |
| 11 | 4.84 | 3.98 | 3.59 | 3.36 | 3.20 | 3.09 | 3.01 | 2.95 | 2.90 | 2.85 | 2.79 | 2.72 | 2.65 | 2.61 | 2.57 | 2.53 |
| 12 | 4.75 | 3.89 | 3.49 | 3.26 | 3.11 | 3.00 | 2.91 | 2.85 | 2.80 | 2.75 | 2.69 | 2.62 | 2.54 | 2.51 | 2.47 | 2.43 |
| 13 | 4.67 | 3.81 | 3.41 | 3.18 | 3.03 | 2.92 | 2.83 | 2.77 | 2.71 | 2.67 | 2.60 | 2.53 | 2.46 | 2.42 | 2.38 | 2.34 |
| 14 | 4.60 | 3.74 | 3.34 | 3.11 | 2.96 | 2.85 | 2.76 | 2.70 | 2.65 | 2.60 | 2.53 | 2.46 | 2.39 | 2.35 | 2.31 | 2.27 |
| 15 | 4.54 | 3.68 | 3.29 | 3.06 | 2.90 | 2.79 | 2.71 | 2.64 | 2.59 | 2.54 | 2.48 | 2.40 | 2.33 | 2.29 | 2.25 | 2.20 |
| 16 | 4.49 | 3.63 | 3.24 | 3.01 | 2.85 | 2.74 | 2.66 | 2.59 | 2.54 | 2.49 | 2.42 | 2.35 | 2.28 | 2.24 | 2.19 | 2.15 |
| 17 | 4.45 | 3.59 | 3.20 | 2.96 | 2.81 | 2.70 | 2.61 | 2.55 | 2.49 | 2.45 | 2.38 | 2.31 | 2.23 | 2.19 | 2.15 | 2.10 |
| 18 | 4.41 | 3.55 | 3.16 | 2.93 | 2.77 | 2.66 | 2.58 | 2.51 | 2.46 | 2.41 | 2.34 | 2.27 | 2.19 | 2.15 | 2.11 | 2.06 |
| 19 | 4.38 | 3.52 | 3.13 | 2.90 | 2.74 | 2.63 | 2.54 | 2.48 | 2.42 | 2.38 | 2.31 | 2.23 | 2.16 | 2.11 | 2.07 | 2.03 |
| 20 | 4.35 | 3.49 | 3.10 | 2.87 | 2.71 | 2.60 | 2.51 | 2.45 | 2.39 | 2.35 | 2.28 | 2.20 | 2.12 | 2.08 | 2.04 | 1.99 |
| 21 | 4.32 | 3.47 | 3.07 | 2.84 | 2.68 | 2.57 | 2.49 | 2.42 | 2.37 | 2.32 | 2.25 | 2.18 | 2.10 | 2.05 | 2.01 | 1.96 |
| 22 | 4.30 | 3.44 | 3.05 | 2.82 | 2.66 | 2.55 | 2.46 | 2.40 | 2.34 | 2.30 | 2.23 | 2.15 | 2.07 | 2.03 | 1.98 | 1.94 |
| 23 | 4.28 | 3.42 | 3.03 | 2.80 | 2.64 | 2.53 | 2.44 | 2.37 | 2.32 | 2.27 | 2.20 | 2.13 | 2.05 | 2.01 | 1.96 | 1.91 |
| 24 | 4.26 | 3.40 | 3.01 | 2.78 | 2.62 | 2.51 | 2.42 | 2.36 | 2.30 | 2.25 | 2.18 | 2.11 | 2.03 | 1.98 | 1.94 | 1.89 |
| 25 | 4.24 | 3.39 | 2.99 | 2.76 | 2.60 | 2.49 | 2.40 | 2.34 | 2.28 | 2.24 | 2.16 | 2.09 | 2.01 | 1.96 | 1.92 | 1.87 |
| 30 | 4.17 | 3.32 | 2.92 | 2.69 | 2.53 | 2.42 | 2.33 | 2.27 | 2.21 | 2.16 | 2.09 | 2.01 | 1.93 | 1.89 | 1.84 | 1.79 |
| 40 | 4.08 | 3.23 | 2.84 | 2.61 | 2.45 | 2.34 | 2.25 | 2.18 | 2.12 | 2.08 | 2.00 | 1.92 | 1.84 | 1.79 | 1.74 | 1.69 |
| 60 | 4.00 | 3.15 | 2.76 | 2.53 | 2.37 | 2.25 | 2.17 | 2.10 | 2.04 | 1.99 | 1.92 | 1.84 | 1.75 | 1.70 | 1.65 | 1.59 |
| 120 | 3.92 | 3.07 | 2.68 | 2.45 | 2.29 | 2.18 | 2.09 | 2.02 | 1.96 | 1.91 | 1.83 | 1.75 | 1.66 | 1.61 | 1.55 | 1.50 |
| ∞ | 3.84 | 3.00 | 2.60 | 2.37 | 2.21 | 2.10 | 2.01 | 1.94 | 1.88 | 1.83 | 1.75 | 1.67 | 1.57 | 1.52 | 1.46 | 1.39 |